British Journal of Mathematics & Computer Science
17(3): 1-11, 2016, Article no.BJMCS.24006
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

Decay of a Saw Tooth–profile in Two-phase-flows of Gas-particle Mixture

Kanti Pandey[1*]

[1]*Department of Mathematics and Astronomy, Lucknow University, Lucknow- 226007, India.*

Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/24006
Editor(s):
(1) Andrej V. Plotnikov, Department of Applied and Calculus Mathematics and CAD, Odessa State Academy of Civil Engineering and Architecture, Ukraine.
Reviewers:
(1) El Akkad Abdeslam, CRMEF Fès, Morocco.
(2) Mohammad Nurul Hoque, Cambrian College, Bangladesh.
(3) P. Paul Divakar, Sir C R Reddy Autonomous College, Eluru, India.
Complete Peer review History: http://sciencedomain.org/review-history/15088

Short Research Article

Received: 31st December 2015
Accepted: 8th June 2016
Published: 21st June 2016

Abstract

Aim: To study the length and decay behavior of a saw tooth-profile in Two-Phase-Flows of gas-particle mixture and compare the analytical solution with numerical solution.
Study Design: Progressive-wave approach is used to obtain asymptotic solution of the non linear system of partial differential equations governing two-phase flows of gas-particle-mixture in reacting gases, which governs the growth and decay of acceleration front. In preparation of graphs origin 7.5 is applied.
Place of Study: Department of Mathematics & Astronomy, Lucknow University, Lucknow-226007, India.
Methodology: Analytical and numerical method **(**Runge-Kutta method of fourth order) is used.
Results: The evolution equation governs the growth and decay of acceleration front and its analytical solution is obtained. With help of this evolution equation length and decay behavior of saw –tooth profile is investigated. These results are compared numerically with help of Runge-Kutta method of fourth order. Numerical results shows that the applied numerical method is in good agreement for length of Saw-Tooth profile for cylindrical case. In case of decay behavior it is in good agreement with analytical one for plane and cylindrical case.
Conclusion: We conclude that the applied method is in good agreement for length of Saw-Tooth profile of cylindrical case and for decay behavior of Saw-Tooth profile for plane and cylindrical case.

**Corresponding author: E-mail: pandey_kanti@yahoo.co.in;*

Keywords: Saw-tooth; two-phase; reacting mixture.

1 Introduction

Studies of non-linear waves by using the progressive wave theory have been carried out by several authors. Germain [1] reviewed the theory of progressive wave for wide applications in several fields. By using asymptotic expansion method Varley and Cumberbatch [2] have studied the finite-amplitude, radically symmetric and isentropic waves in fluids. Seymour and Varley [3] have studied high frequency periodic disturbances in dissipative systems and their results were applied to non-linear transmission lines, non-linear dielectrics and non-linear waves in a reacting gas mixture. Through asymptotic analysis approach, Fusco [4] concluded that for a wave motion described by non homogeneous quasi-linear-hyperbolic system, coupling between non-linearity and dissipative effects can be established. Fusco and Engelbreckt [5] have presented asymptotic analysis of non linear waves in rate-dependent media to study the high and low frequency wave processes and have obtained an evolution equation for a visco-elastic media. Sharma et al. [6] and Shukla et al. [7] have applied the progressive wave approach to study the decay behavior of a saw-tooth profile in magneto fluid and chemically reacting gases respectively.

Studies of non-linear effects on the wave propagation have been extensively carried out by Jeffery and Taniuti [8], Whitham [9], Courant and Friedrichs [10]. In recent technological advancement in different branches of engineering and science, compressible flows of dusty gas has attracted the attention of many investigators [11-19]. The study of wave propagation in a mixture of gas and dust particle has many engineering applications such as flow in rocket, nuclear-reactors, fuel-spray, air pollution and numerous application in underground applications [20,21].

It is not always possible to exactly solve non-linear differential equation, hence numerical solution plays an important role to deal with such problems. Numerical calculations leads to approximate results, thus there is a difference between the exact and computed values, known as rounding error. If this error is minimal then this method is suitable. References [22-25] shows some numerical solutions of non- linear waves.

Having such important applications of two-phase flows of gas particle mixture, in present paper using asymptotic analysis, analytical and numerical solution of decay of saw-tooth profile in two-phase flow of reacting gas is investigated. Throughout this article we have considered the two-phase flows model considered by Rudinger [26], when particle volume fraction is negligible.

1.1 Basic equations

The basic equations governing the one dimensional motion of mixture of a reacting gas and a large number of small dust particles of uniform spherical shape, when viscosity and heat conductivity of gas are neglected are given by Rudinger [26].

$$(D\rho/ Dt)+\rho u_{,x}+(\rho u\, m)/x=0, \quad (1)$$

$$(D u / Dt) + \{1/\rho(\eta +1)\}\, p_{,x} = 0, \quad (2)$$

$$(Dh / Dt) - (1/\rho)(Dp/ Dt) = 0, \quad (3)$$

where p, u, ρ and h denote pressure, velocity, density and enthalpy of the mixture respectively. $(D/Dt)=\partial/\partial t + u\partial/\partial x$ and a comma followed by an index denote partial derivative with respect to that index, m = 0, 1, 2 for planer, cylindrical and spherical symmetry. Considering moderate particle loading case considered by Rudinger [26] η(mass flow ratio for equilibrium case) and density ρ of mixture are defined as:

$\rho= \rho_p + \rho_g$, ρ_p, ρ_g being density of particle material and gas respectively.

$\eta = \varphi / (1- \varphi)$, when gas and particle velocity are equal, φ being mass fraction of particle.

We assume that the time characterizing a macroscopic change under study is long compared to the time of establishing Boltzmann's distribution among translational degrees of freedom of the molecules in the mixture, thus rate equation is given by:

$$(Dq / Dt) = \dot{q}, \tag{4}$$

where, q is assumed to be a known function of p, ρ and q Vincenti & Kruger[27] and canonical equation of state is given by,

$$h = h(p,S,q), \tag{5}$$

S being entropy of medium.

Using Gibbs relation,

$$T\, dS = dh - (1/\rho)dp + A dq, \tag{6}$$

and canonical equation of state, we have following equivalent system of governing equations:

$$p_{,t} + u\, p_{,x} + \rho\, a^2_f \{u_{,x} + (mu)/x\} + a^2_f [\{ (A \rho_{,S}) / T\} + \rho_{,q}]\, \dot{q} = 0, \tag{7}$$

$$u_{,t} + u u_{,x} + p_{,x}\{1/\rho(\eta+1)\} = 0, \tag{8}$$

$$S_{,t} + u\, S_{,x} - (A\dot{q}) / T = 0, \tag{9}$$

$$q_{,t} + u\, q_{,x} - \dot{q} = 0, \tag{10}$$

where T is temperature, A is the affinity of the internal transformation characterized by variable q and, $a^2_f = \{- h_{,\rho}/\rho(\eta+1)\, (h_{,p} - 1/\rho)\}$ being square of resulting sound speed. Equations (7-10) in dimensionless form can be written as

$$U^i_{,t} + C^{ij}\, U^i_{,x} + D^i = O \tag{11}$$

U^i being Column vector with four components p, u, S, q, C^{ij} is 4X4 matrix, D^i is a column vector with four components which can be obtained from equations (7-10) by inspection. System (11) is hyperbolic in nature and matrix C^{ij} of equation (11) has four real eigenvalues $(u \pm a_f)$ and u twice. Left and right eigenvectors corresponding to eigenvalues $(u + a_f)$ are given by,

$$L^1 = [1 , \rho a_f , 0 , 0],$$
$$R^1 = [1 , 1/(\rho a_f) , 0 , 0]^T,$$

where superscript T denotes transposition.

2 Methodology

a – Progressive wave solution: To find an asymptotic solution of system (11), let us consider asymptotic expansion for U^i in the following form:

$$U^i(x, t) = U^i_0 + \epsilon\, U^i_1 (x ,t, \xi) + \epsilon^2\, U^i_2 (x, t , \xi) + O(\epsilon^n), \tag{12}$$

where, $U^i{}_0$, is a known constant state solution of equation (11) such that,

$D^i(U_0) = 0$ and

$\epsilon = \tau_{ch} / \tau_\alpha << 1$ is a small parameter, τ_{ch}- characterizes time scale for medium and τ_α is attenuation time characterizing dissipative mechanism.

Introducing Taylor's expansion of C^{ij} and D^i in neighborhood of the known constant solution $U^i{}_0$ and using asymptotic expansion given by equation (12) in equation (11) and collecting coefficients of constant term and ϵ, we have following set of equations:

$$(C^{ij}{}_0 - \lambda \delta^i{}_j) U^j{}_{1,\xi} = 0 \tag{13}$$

$$(C^{ij}{}_0 - \lambda \delta^i{}_j) U^j{}_{2,\xi} + f^{-1}{}_{,x}\{(U^i{}_{1,t} + C^{ij}{}_0(U^j{}_{i,x})\} + U^k{}_1(C^{ij}{}_{,Uk})_0 (U^j{}_{1,\xi}) + f_{,x}{}^{-1} U^k{}_1 (D^i{}_{,Uk})_0 = 0, \tag{14}$$

where, $\xi = f(x, t) / \epsilon$ is fast variable, f(x,t) being Phase Function characterizing wave front, $\lambda = -f_{,t} / f_{,x}$ and $\delta^i{}_j$ being kronecker delta.

Equation (13) shows that $U^i{}_{1,\xi}$ is collinear to a right eigenvector $R^i{}_0$. Thus, $U^i{}_1$ can be written as

$$U^i{}_1(x, \xi, t) = g(x, t, \xi) R^i{}_0 + S^i (x, t), \tag{15}$$

$g(x, t, \xi)$ being wave amplitude which we have to determine and S^i are constants of integration, which are not of a progressive wave nature and can be taken as zero.

The phase equation and evolution function g are given by:

$$f(x,t) = x - x_0 - a_{f0} t \tag{16}$$

and,

$$\partial g/\partial \tau + E_0 g \partial g/\partial \xi + Q_0 g = 0, \tag{17}$$

where,

$$E_0 = [(\Gamma + 1) / 2\rho a_f]_0,$$

$$Q_0 = (1/ 2\tau_0)[\{ (a^2{}_{f0} / a^2 e_0) - 1 \} + \{(m a_{f0})/ 2 (x_0 + a_{f0} t)\}],$$

a_f is frozen speed of sound, $\partial / \partial\tau = \partial/ \partial t + a_{f0}\, \partial/ \partial x$ is Ray derivative, $\Gamma = 1 + \rho (\partial a^2{}_f / \partial p) = \gamma_M (1 + \zeta \eta) / (1 + \gamma_M \zeta \eta)$, ratio of specific heats of mixture, $\gamma_M = \Omega (1 + \zeta \eta) / (1 + \Omega \zeta \eta)$, $\Omega = (4+q)/3$, for ideal case $\Omega = \gamma$, η being mass loading ratio, ζ is relative specific heat (c/ c_p), c being specific heat of particle material, c_p is specific heat of gas at constant pressure and subscript zero implies that quantity is evaluated at equilibrium which means:

$$\Gamma_0 = \Gamma (p_0, S_0, q_0).$$

b – Acceleration wave front: In order to consider an acceleration wave front described by the curve $f(x, t) = 0$, u may be represented by,

$u = \epsilon\, u_1 (x, t, \xi) + O(\epsilon^2)$ where,

$u_1 = 0$, for $\xi < 0$

and

$u_1 = O(\xi)$ for $\xi > 0$.

Thus in view of equation (15), $g(x, t, \xi)$ is given by,

$$\left.\begin{aligned} &g(x, t, \xi) = 0 \text{ for } \xi < 0 \\ &g(x,t,\xi) = \xi f(x, t) + O(\epsilon^2) \text{ for } \xi > 0, \end{aligned}\right\} \tag{18}$$

where, $f(x, t) = (\sigma/a_{f0})$ and $\sigma = [u,_x]$ denotes the jump in the velocity gradient across the acceleration front and may be described as the wave amplitude of acceleration front.

Substituting $g(x, \xi, t)$ from equation (18) into equation (17), we have following type of Bernoulli equation:

$$d\sigma/dt + \sigma[B + \{(m\, a_f / 2(x + a_f t)\}]_0 + \Lambda_0 \sigma^2 = 0, \tag{19}$$

where, $\Lambda_0 = (\Gamma + 1)/2$ and

$$B_0 = (1/2\tau_0)\{(a^2{}_{f0} / a^2 e_0) - 1\}.$$

Equation (19) governs growth and decay of acceleration front and its solutions are:

$$\sigma = [\sigma^* \exp(-B_0 t)] / [1 + \{(\Lambda_0 \sigma^*)/B_0\}\{1 - \exp(-B_0 t)\}] \quad \text{for } m = 0 \tag{20}$$

$$\sigma = [\sigma^* \exp(-B_0 t)(x_0 + a_{f0} t)^{(-1/2)}] / [(x_0)^{(-1/2)} + (\Lambda_0 \sigma^*) \{\pi/(B_0 a_{f0})\}^{1/2} \exp(B_0 x_0/a_{f0}) \{\mathrm{erf}(B_0 x/a_{f0})^{1/2} - \mathrm{erf}(B_0 x_0/a_{f0})^{1/2}\}] \quad \text{for } m = 1 \tag{21}$$

$$\sigma = [\sigma^* \exp(-B_0 t)(x_0 + a_{f0} t)^{(-1)}] / [(x_0)^{(-1)} + \{(\Lambda_0 \sigma^*)/a_{f0}\} \exp(B_0 x_0/a_{f0}) \{\mathrm{Ei}(B_0 x_0/a_{f0})^{1/2} - \mathrm{Ei}(B_0 x/a_{f0})^{1/2}\}] \quad \text{for } m = 2 \tag{22}$$

where,

$$\mathrm{erf}(x) = (2/\pi^{1/2}) \int_0^x \exp(-t^2)\,dt, \text{ and}$$

$$\mathrm{Ei}(x) = \int_0^x \{(1 - e^{(-t)})/t\}\,dt - \ln x - r,$$

are error and exponential functions respectively. In deriving these results help of reference [28] is taken. $\sigma^* \neq 0$ is the initial value of σ. As B_0 and Λ_0 are positive constant solution of equations (20) to (21) shows that all expansion waves ($\sigma^* > 0$) decreases exponentially and will be damped out ultimately. On the other hand compressive waves ($\sigma^* < 0$) will decay out and terminate into a shock-wave and if in case $\sigma^* = B_0/\Lambda_0$, velocity at wave- front will remain constant results similar to [29,30].

c- **Decay of saw-tooth profile:** Now we consider a physical situation when a compressive wave terminates into a weak shock front propagating into the medium at rest, followed by an expansion wave front. Such physical situation may be described in the form of a saw – tooth profile. As time passes, the compressive part of the initial wave steepness into a weak -shock due to non-linear effect, but the expansion part propagates in the form of an expansion wave following the shock with a speed of propagation a_{f0}. The associated shock part propagates at a faster speed $G > a_{f0}$ and location of the shock at any time t is given by.

$$x_s(t) = x_0 + a_{f0} t + L(t), \tag{23}$$

L(t) being the length of the saw – tooth profile at any time t.

The velocity G in this case is given by,

$$G = d\,x_s(t)/dt = a_{f0} + dL/dt. \qquad (24)$$

From weak- shock conditions we have,

$$G = a_{f0} + u_1(\Gamma + 1)/4, \qquad (25)$$

where $u_1 = a_{f0}\,\delta$, $\delta = [\rho]/\rho_0$ denotes shock-strength parameter.

The particle velocity u at the rear of the weak-shock heading the saw-tooth profile can be expressed as,

$$u_1 = \sigma\,L(t). \qquad (26)$$

From equations (23) to (25) we have,

$$(dL/dt) = \{\sigma\,L(t)\,\Lambda_0\}/2. \qquad (27)$$

Integrating equation (27) we have,

$$L/L^* = [1 + \{(\Lambda_0\sigma^*)/B_0\}\{1 - \exp(-B_0 t)\}]^{1/2} \quad \text{for} \quad m = 0, \qquad (28)$$

$$L/L^* = [1 + (\Lambda_0\sigma^*(x_0)^{(1/2)}) \{\pi/(B_0 a_{f0})\}^{1/2} \exp(B_0 x_0/a_{f0}) \{\mathrm{erf}(B_0 x/a_{f0})^{1/2} - \mathrm{erf}(B_0 x_0/a_{f0})^{1/2}\}]^{(1/2)} \quad \text{for} \quad m = 1 \qquad (29)$$

$$L/L^* = [1 + \{(\Lambda_0\sigma^* x_0)/a_{f0}\} \exp(B_0 x_0/a_{f0}) \{\mathrm{Ei}(B_0 x_0/a_{f0})^{1/2} - \mathrm{Ei}(B_0 x/a_{f0})^{1/2}\}]^{1/2} \quad \text{for } m = 2, \qquad (30)$$

Applying equations (20 – 22) and (28 – 30) in equation (26) we have,

$$u = [L^*\sigma^* \exp(-B_0 t)]/[1 + \{(\Lambda_0\sigma^*)/B_0\}\{1 - \exp(-B_0 t)\}]^{1/2} \quad \text{for} \quad m = 0, \qquad (31)$$

$$u = [\sigma^* L^* \exp(-B_0 t)\{x_0\,(x_0 + a_{f0} t)^{(-1/2)}\}]/[1 + \{\Lambda_0\sigma^*(x_0)^{(1/2)}\} \{\pi/(B_0 a_{f0})\}^{1/2} \exp(B_0 x_0/a_{f0}) \{\mathrm{erf}(B_0 x/a_{f0})^{1/2} - \mathrm{erf}(B_0 x_0/a_{f0})^{1/2}\}]^{1/2} \quad \text{for} \quad m = 1, \qquad (32)$$

$$u = [\sigma^* L^* \exp(-B_0 t)\{x_0\,(x_0 + a_{f0} t)^{-1}\}]/[1 + \{(\Lambda_0\sigma^* x_0)/a_{f0}\} \exp(B_0 x_0/a_{f0})\{\mathrm{Ei}(B_0 x_0/a_{f0})^{1/2} - \mathrm{Ei}(B_0 x/a_{f0})^{1/2}\}]^{1/2} \quad \text{for} \quad m = 2, \qquad (33)$$

3 Results and Discussion

Progressive-wave approach is used to obtain asymptotic solution of the non-linear system of partial-differential equations governing two-phase flows of gas-particle- mixture in case of reacting gases. For wave amplitude a Burger type equation is obtained from which Bernoulli type evolution equation is derived. This evolution equation governs the growth and decay of acceleration front and its analytical solution is obtained. Next for saw-tooth profile length and decay behavior is investigated. These results are compared numerically with help of Runge-Kutta method of fourth order. Numerical results shows that the applied numerical method is in good agreement for length of Saw-Tooth profile for cylindrical case. In case of decay behavior it is in good agreement with analytical one for plane and cylindrical case. In preparation of graphs origin 7.5 is applied.

Figs. 1-3, shows the non-equilibrium effects on the length of saw-tooth profile for planer, cylindrical and spherical waves. Figs. 4-6 shows decay behaviour of saw-tooth profile under non-equilibrium effects.

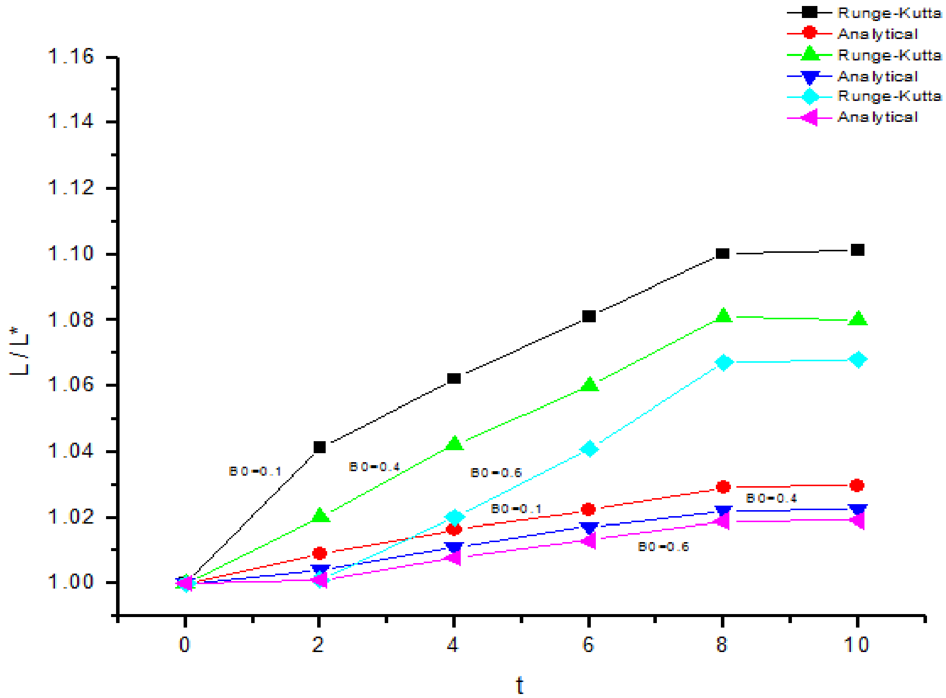

Fig. 1. Non-equilibrium effect on the length of a saw-tooth profile for plane wave

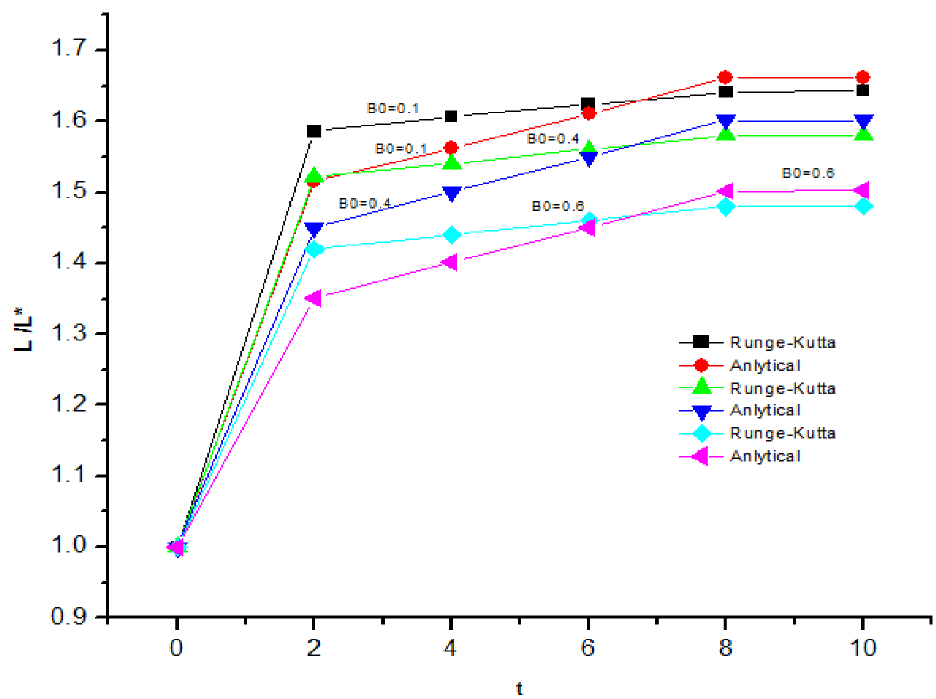

Fig. 2. Non-equilibrium effects on the length of a saw-tooth profile for cylindrical wave

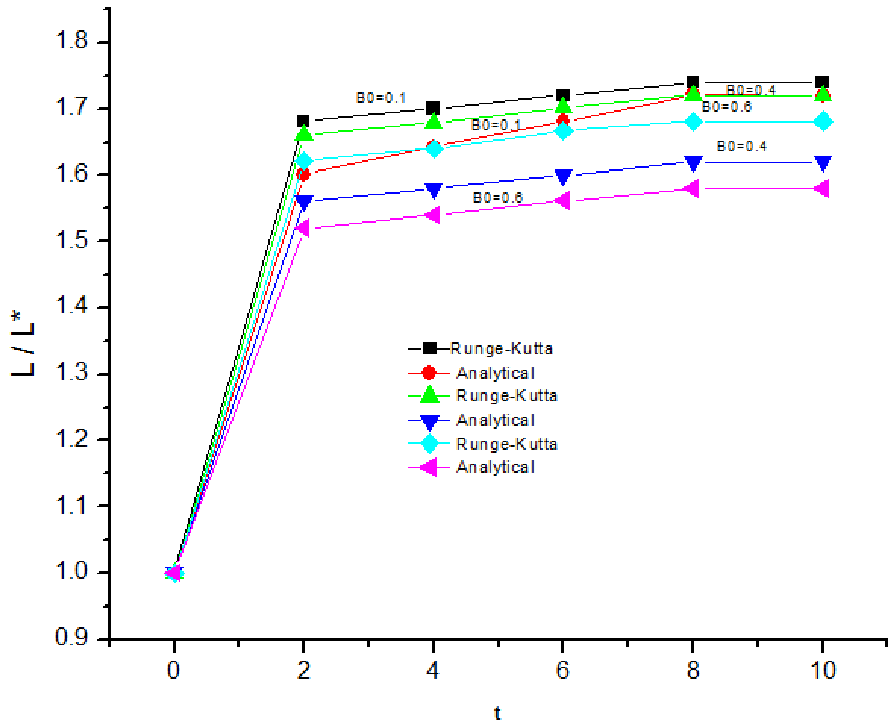

Fig. 3. Non-equilibrium effects on the length of a saw-tooth profile for spherical wave

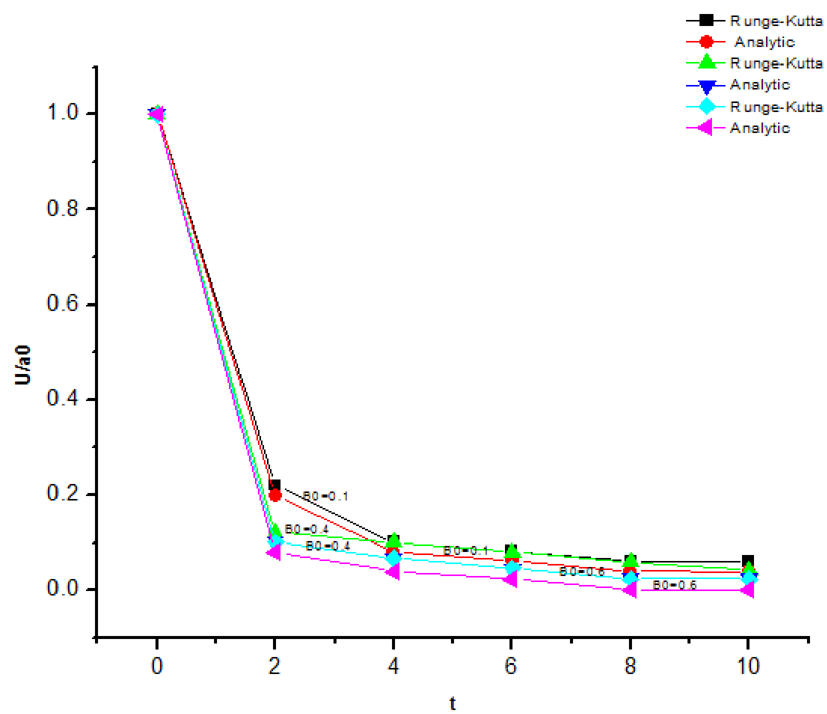

Fig. 4. Decay of saw-tooth profile for plane wave under non-equilibrium effect

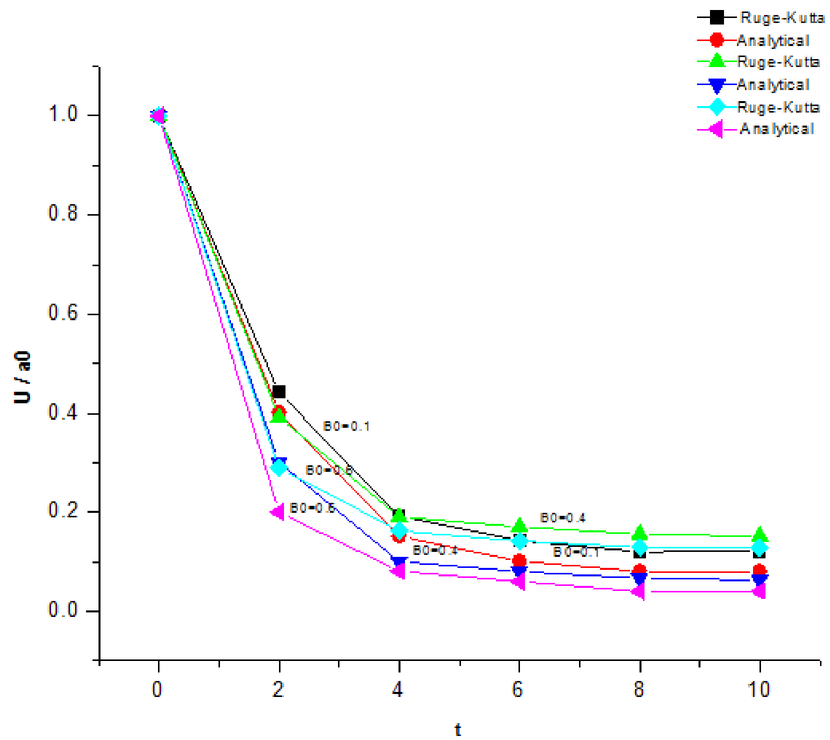

Fig. 5. Decay of saw-tooth profile for cylindrical wave for non equilibrium effect

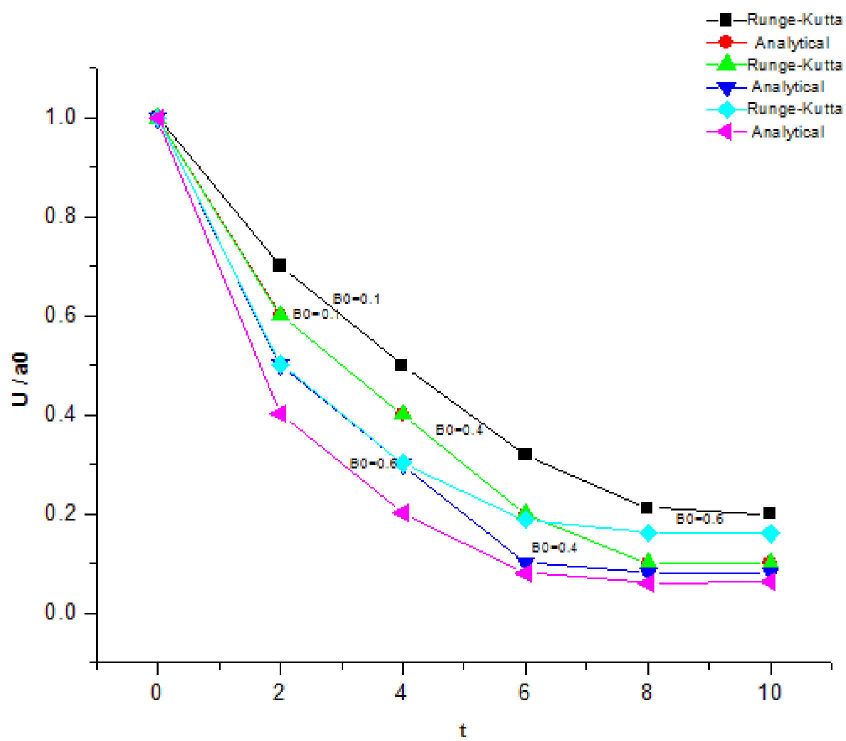

Fig. 6. Decay of saw-tooth profile of spherical wave under effect of non-equilibrium effect

4 Conclusion

Figs. 1-3, shows the non-equilibrium effects on the length of saw-tooth profile for planer, cylindrical and spherical waves and is concluded that for cylindrical waves numerical results are in good agreement with analytical result. Figs. 4-6 shows decay behaviour of saw-tooth profile under non-equilibrium effects and results show that for planer and cylindrical case numerical method has a good agreement with analytical one.

Acknowledgements

Author is grateful to U.G.C. for providing financial assistance in preparation of this paper.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Germain P. Progressive waves. Jber DGLR. 1972;11-29.

[2] Varley E, Cumberbatch E. Non-linear high frequency sound waves. J. Inst. Maths. Applic. 1966;2: 133-143.

[3] Seymour BR, Varley E. High frequency, periodic disturbances in dissipative system, I, Small amplitude, finite rate theory. Proc. R. Soc. Lond. 1970;314A:387-415.

[4] Fusco D. Some comments on waves described by non-homogeneous quasi-linear first order hyperbolic system. Meccanica. 1982;17:128-137.

[5] Fusco D, Engelbreckt J. The asymptotic analysis of non-linear waves in rate-dependent media II. Nuovo Cim. 1984;80B:49-61.

[6] Sharma VD, Singh LP, Ram R. The progressive wave approach analyzing the decay of a saw-tooth profile in magneto-gas dynamics. The Phys. Fluids. 1987;30:1572 -1574.

[7] Shukla P, Sharma RR, Singh LP. Decay of a saw-tooth profile in chemically reacting gases. Int. J. Engng. Sci. 1994;32(3):527-533.

[8] Jeffery A, Taniuti T. Non-linear wave propagation. Academic Press, New York; 1964.

[9] Whitham GB. Linear and non-linear waves. Wiley, New York; 1974.

[10] Courant R, Friedrich KO. Supersonic flow and shock-waves. Springer Varley, New York; 1948.

[11] Kriebel AR. Analysis of normal shock-waves in particle laden gas. J. Basic Eng. 1964;86:655-663.

[12] Pai SI. Two phase flows. Vieweg, tracts in Pure Appl. Phys. Vieweg Verlag Braunsch Weing. 1977;3(chapter v):116-167.

[13] Pandey K, Saxsena M. Effect of finite volume fraction on curvature of a reflected shock in gas particle mixture. Jour. Math. Phy. Sci. 1992;26(1):61-76.

[14] Pandey K, Saxsena, Shukla MK. Propagation of sonic waves in gas particle mixture. Journal of Mathematical and Physical Sciences. 1996;30(2):77-88.

[15] Pandey K, Vaish AK. Waves in two phase flows. Advanced Studies in Pure Mathematics Asymptotic Analysis and Singularities. 2007;47(1):303-320.

[16] Pandey K, Singh K. Strong explosions in two phase mixture of radiating gases. Journal of International Academy of Physical Sciences. 2009;13(3):1-12.

[17] Pandey K, Singh K. Non self similar solution of shock waves in gas particle mixture. Rev. Bull. Cal Math. Soc. 2011;19(1):69-82.

[18] Pandey K, Verma P. Certain aspects of weak shocks in steady non equilibrium two phase flows along a curved wall. International Journal of Mathematical Analysis (IJMA). 2012;6(7):311-319.

[19] Pandey K, Verma P. Shock formation down a non- uniform tube in two phase flows. Journal of Mathematics Research. 2013;5(3):17-25.

[20] Mcqueen RG. Shock waves in condensed medium Eliezer S. and Rice R.A. (Eds.), High pressure equation of state, theory and application. North Holland, Amsterdam; 1991.

[21] Nagayama K. Shock wave interaction in solid material - Sawaoka AB. (Ed.), Shock. Waves in Material Science. Springer, Berlin; 1993.

[22] Pandey K, Verma L, Verma AK. On a finite difference scheme for Burger's equation. Applied Mathematics and Computation (Elsevier). 2009;215:2206-2214.

[23] Pandey K, Verma L. A note on Crank-Nicolson scheme for Burgers equation. Applied Mathematics (Scientific Research Publishing), USA. 2011;2:883-889.

[24] Pandey K, Verma L, Verma AK. L-stable Simpson's 3/8 rule and Burger's equation. Applied Mathematics and Computation (Elsevier). 2011;218:1342-1352.

[25] Pandey K, Verma L, Verma AK. Du Fort-Frankel finite difference scheme for Burgers equation. Arab. J. Maths. 2013;2(1):91-101.

[26] Rudinger G. Wave propagation in suspension of solid particles in gas flow. Applied Mechanics Review. 1973;26:273-279.

[27] Vincenti WG, Kruger CH. Introduction to physical gasdynamics. Wiley, New York; 1965.

[28] Milton A, Stegun IA. (Eds.). Hand book of mathematical functions with formulas, graphs and mathematical tables; 1970.

[29] Pandey K, Saxsena M. Weak non linear waves in dissociating gas. Indian Jour. Pure. Appl. Math. 1991;22(6):513-523.

[30] Pandey K, Verma P. Certain aspects of weak shocks in steady non equilibrium two phase flows along a curved wall. International Journal of Mathematical Analysis (IJMA). 2012;6(7):311-319.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://sciencedomain.org/review-history/15088