Thermal and Fluid Effects of Non-Newtonian Water-based Nanofluids on the Free Convection Flow between Two Vertical Planes

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Authors' contributions

This work was carried out in collaboration between all authors. Authors YR, RY and SEG designed the study, performed the statistical analysis and the literature searches and wrote the protocol. Author DDG managed the analysis of the study and wrote the protocol, and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

Research Article

ABSTRACT

A nonlinear analytical technique (Homotopy Perturbation Method) has been developed to assess the effects of several nanofluids characteristics on the free convective heat transfer to the power-law non-Newtonian flow between two infinite parallel vertical flat plates. A numerical method (Runge-Kutta) also has been done in order to show the accuracy of Homotopy Perturbation Method (HPM), then several graphs have been drawn which show the effects of nanoparticle volume fraction, kind of nanofluids, Eckert number and dimensionless non-Newtonian viscosity on the velocity and temperature profiles of problem. Finally, for various values of dimensionless non-Newtonian viscosity, the effects of different value of nanofluid nanoparticle volume fraction on the Heat transfer coefficient and skin friction are presented and discussed.

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1. INTRODUCTION

Natural convection flow is a well studied problem having a vital role in many engineering applications such as heat exchangers, insulation, building ventilation, refrigeration, solar energy collection, petroleum reservoirs, nuclear waste repositories and etc. Ostrach [1] and Khalifa [2] presented a review of heat transfer due to natural convection. Despite the fact that the importance of non-Newtonian fluids in modern technology and industries processes is recognized in a wide range of studies, only a few studies have been published on non-Newtonian nanofluids. B K Jha and A O Ajibade presented a transient motion of a viscous and incompressible fluid between two infinite vertical parallel plates due to natural convection currents occurring as a result of application of isothermal and adiabatic conditions on the plates. They used the method of Laplace transform to solve the problem [3]. Natural convection heat and mass transfer of nanofluids over a vertical plate embedded in a saturated Darcy porous medium subjected to surface heat and nanoparticle fluxes is analysied by Noghrehabadi et al. [4]. they carried out the numerical solution in two steps. The governing partial differential equations were firstly simplified into a set of highly coupled nonlinear ordinary differential equations by appropriate similarity variables, and then numerically solved by the finite difference method. Rashad et al. [5] performed an analysis to study the effect of uniform transpiration velocity on free convection boundary-layer flow of a non-Newtonian fluid over a permeable vertical cone embedded in a porous medium saturated with a nanofluid [5].

An exact analysis of the natural convection in unsteady Couette flow of a viscous incompressible fluid confined between two vertical parallel plates in the presence of thermal radiation was performed by M. Narahar [6]. A numerical analysis was performed to examine the heat transfer of colloidal dispersions of Au nanoparticles in water by Primo Ternik et al. [7]. They reported exact numerical results showing clearly that the average Nusselt number is a growing function of both volume fraction of Au nanoparticles and Rayleigh number. Niu J et al. [8] studied the slip-flow and heat transfer of a non-Newtonian nanofluid in a microtube by means of theoretical method [8]. In their research, the power-law rheology was adopted to describe the non-Newtonian characteristics of the flow, in which the fluid consistency coefficient and the flow behavior index depend on the nanoparticle volume fraction. The volumetric flow rate and local Nusselt number were calculated for different values of nanoparticle volume fraction and slip length. They presented that an increase in the nanoparticle volume fraction will lead to a decrease in the flow rate at a small pressure gradient and constant slip length, but as the pressure gradient becomes large enough, the flow rate increases with the increase of the nanoparticle volume fraction. They also concluded that this important phenomenon will be observed when the radius of the tube shrinks to micrometer scale. They studied the effect of thermal boundary condition on the thermally fully developed heat transfer of the nanofluid. Despite the fact that most of thermal and fluid phenomenons are expressed by nonlinear equations, only a few methods are able to solve them. One of these methods is HPM which was introduced by J. H. He [9]. This method has been developed by many researchers [10–13] to solve different types of non-linear and nonhomogeneous differential equations of heat and fluid flow problems. This method also gives an acceptable solution of natural convection of a non-Newtonian fluid as it will be observed in this paper.

2. DEFINITION OF THE PROBLEM

Simple schematic geometry of present study is shown in Fig. 1.
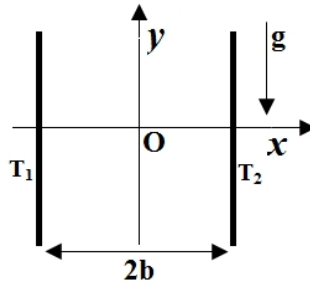
Fig. 1. Schematic model of present problem

A non-Newtonian fluid flows between two vertical flat plates with 2b distance from each other. Constant temperatures T_2 and T_1 are assumed to be the temperature of walls at x=+b and x= - b, respectively, where $T_2 > T_1$. This difference in temperature causes the fluid near the wall at x=- b to rise and the fluid near the wall at x=+b to drop down.

The equation of motion is [16]:

$$\mu \frac{d^2 v}{dx^2} + 6\beta^3 (\frac{dv}{dx})^2 \frac{d^2 v}{dx^2} + \rho_0 \gamma (T - T_m) g = 0 \qquad (1)$$

And the energy equation as follows:

$$k \frac{d^2 T}{dx^2} + \mu (\frac{dv}{dx})^2 + 2\beta^3 (\frac{dv}{dx})^4 = 0 \qquad (2)$$

The fluids are water-based nanofluids containing TiO_2, CuO, Al_2O_3, Cu and Ag. It is also assumed that the nanoparticles and the Fluid phase are in thermal equilibrium and there is no slip between them. Table 1 shows the thermo-physical properties of water and the elements Cu, TiO2, CuO, Ag, and Al_2O_3.

Table 1. Thermophysical properties

	$\rho(kg/m^3)$	$C_p(J/kgk)$	K (w/m.k)	$\beta^{\times 10^5}(k^{-1})$
Pure Water	997.1	4179	0.613	21
Cu	8933	385	401	1.67
TiO_2	4250	686.2	8.9538	0.9
CuO	535.6	6500	20	57.45
Ag	10500	235	429	1738.6
Al_2O_3	3970	765	40	0.85

Maxwell-Garnetts (MG) approximation can be used to find the effective viscosity of the nanofluid (Khanafer et al.) [14] as:

$$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \qquad (3)$$

The effective density of the fluid is given as:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \tag{4}$$

The thermal conductivity of the nanofluid is approximated as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles and is given by Brinkman [15]:

$$A_1 = \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + \varphi(k_f - k_s)} \tag{5}$$

Where φ is the nanoparticle volume fraction. Introducing the following dimensionless variables based on the study of Rajagopal and Na [16]:

$$V = \frac{v}{V_0} \qquad \eta = \frac{x}{b} \qquad \theta = \frac{T - T_m}{T_1 - T_2} \tag{6}$$

Then, momentum, and energy equations, in non-dimensional form, in Cartesian coordinates can be reduced to the following forms:

$$\frac{d^2V}{d\eta^2} + 6\delta(1-\varphi)^{2.5}(\frac{dV}{d\eta})^2 \frac{d^2V}{d\eta^2} + \theta = 0 \tag{7}$$

$$\frac{d^2\theta}{d\eta^2} + Ec \times \mathrm{Pr} \times \frac{(1-\varphi)^{-2.5}}{A_1}(\frac{dV}{d\eta})^2 + 2\delta Ec \times \mathrm{Pr} \times \frac{1}{A_1}(\frac{dV}{d\eta})^4 = 0 \tag{8}$$

Where the boundary condition, Prandtl number (Pr), Eckert number (Ec) and dimensionless non-Newtonian viscosity (δ) are given as:

$$V(\eta = -1) = 0, \quad \theta(\eta = -1) = -0.5, \quad V(\eta = 1) = 0, \quad \theta(\eta = 1) = 0.5 \tag{9}$$

$$\mathrm{Pr} = \frac{\mu_f(\rho C_p)_f}{\rho_f k_f} \quad , \quad Ec = \frac{\rho V_0^2}{(\rho C_p)_f(\theta_1 - \theta_2)} \quad , \quad \delta = \frac{6\beta V_0^2}{\mu_f b^2} \tag{10}$$

The average Heat transfer coefficient (h) and skin friction (S) are defined as [16]:

$$h \approx \frac{d\theta}{dx}(-1) \quad \text{and} \quad S \approx \frac{dV}{dx}(-1) \tag{11}$$

3. HOMOTOPY PERTURBATION METHOD

In order to demonstrate the HPM method, consider the following equation as follows:

$$A(u) - f(r) = 0 \quad r \in \Omega \quad \text{with} \quad bc : B(u, \frac{\partial u}{\partial n}) = 0 \quad r \in \Gamma$$

$$(12)$$

Where *A* is a general differential operator, *B* a boundary operator, *f* (*r*) a known analytical function and Γ is the boundary of the domain Ω . *A* can be written into two parts, *L* for linear and *N* nonlinear part. Eq. (10) can therefore be defined as follows:

$$L(u) + N(u) - f(r) = 0 \quad r \in \Omega$$

$$(13)$$

Homotopy perturbation structure is introduced as follows:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \quad where \quad v(r, p) : \Omega \times [01] \rightarrow R$$

$$(14)$$

In Eq. (5), $p \in$ [0, 1] is an embedding parameter and u_0 is the first approximation that satisfies the boundary condition. It can be assumed that the solution of Eq. (12) can be written as a power series in *p,* as following:

$$v = v_0 + v_1 p + v_2 p^2 + ...$$

$$(15)$$

The best approximation for solution can be achieved when

$$u = \lim_{p \to 1} (v_0 + v_1 p + v_2 p^2 + ...)$$

$$(16)$$

4. HPM SOLUTION

According to the HPM method, Eq (17) and Eq (18) can be written according to the Eq. (7) as -

$$H(V, p) = (1 - p)(\frac{d^2 V}{d\eta^2}) + p[6\delta(1 - \varphi)^{2.5}(\frac{dV}{d\eta})^2 \frac{d^2 V}{d\eta^2} + \theta]$$

$$(17)$$

And:

$$H(\theta, p) = (1 - p)(\frac{d^2\theta}{d\eta^2}) + p[\frac{d^2\theta}{d\eta^2} + Ec \times \text{Pr} \times \frac{(1 - \varphi)^{-2.5}}{A_1}(\frac{dV}{d\eta})^2 + 2\delta Ec \times \text{Pr} \times \frac{1}{A_1}(\frac{dV}{d\eta})^4]$$

$$(18)$$

According to eq. 6, the momentum and energy equations can be assumed as -

$$f(\eta) = f_0(\eta) + pf_1(\eta) + p^2 f_2(\eta) + p^3 f_3(\eta)$$

$$(19)$$

$$V(\eta) = V_0(\eta) + pV_1(\eta) + p^2 V_2(\eta) + p^3 V_3(\eta)$$

$$(20)$$

By Substituting equations (19) and (20) into Eq. (21) and (22) and collecting all the coefficients with the same rational power of "p":

$$\begin{cases} \dfrac{d^2}{d\eta^2}V_0(\eta) + P\{\dfrac{d^2}{d\eta^2}V_1(\eta) + \theta_0(\eta) + ...\} + p^2\{6\delta^{(1-\phi)2.5}(\dfrac{d}{d\eta}V_0(\eta))^2\dfrac{d^2}{d\eta^2}V_0(\eta) + ...\} + ... \\[4mm] and \\[4mm] \dfrac{d^2}{d\eta^2}\theta_0(\eta) + P\{\dfrac{d^2}{d\eta^2}\theta_1(\eta) + \dfrac{Ec\,\Pr(\dfrac{d}{d\eta}V_0(\eta))^2}{(1-\phi)^{2.5}A_1} + ...\} + p^2\{(\dfrac{d^2}{d\eta^2}\theta_2(\eta))^2...\} + ... \end{cases}$$

$$(21)$$

The coefficients of P^i in eqs. 19 and 20 must be zero in order to find the functions of $V_i(\eta)$ and $\theta_i(\eta)$. It should be noted that, boundary condition for the coefficient of P^0 is:

$$V_0(\eta = -1) = 0, \quad \theta_0(\eta = -1) = -0.5, \quad V_0(\eta = 1) = 0, \quad \theta_0(\eta = 1) = 0.5 \quad (22)$$

Then, by applying this boundary condition, $V_0(\eta)$ and $\theta_0(\eta)$ will be resulted as:

$$V_0(\eta) = \frac{1}{12}\eta^3 - \frac{1}{12}\eta, \quad \theta_0(\eta) = -0.5\eta$$

$$(23)$$

Boundary conditions for the coefficient of P^1, P^2... are:

$$V_{i,i\neq0}(\eta) = 0, \quad \theta_{i,i\neq0}(\eta) = 0, \quad V_{i,i\neq0}(\eta) = 0, \quad \theta_{i,i\neq0}(\eta) = 0 \quad (24)$$

Then, by applying these boundary conditions, $V_1(\eta)$, $\theta_1(\eta)$, $V_2(\eta)$, $\theta_2(\eta)$, $V_3(\eta)$, $\theta_3(\eta)$..., will be obtained. In fact each step helps to find the momentum and energy functions of next step. For instance, using the eq. 21 in the coefficient of P^1:

$$V_1(\eta) = -\frac{1}{2016}(9\delta\sqrt{1-\phi}\phi^2 + 9\delta\sqrt{1-\phi} - 18\delta\sqrt{1-\phi}\phi)\eta^7 + ...$$

$$\theta_1(\eta) = -\frac{1}{10368}\frac{1}{(1-\phi)^{5/2}A_1}(Ec\,\Pr(\frac{1}{90}(81\delta\sqrt{1-\phi}\phi^2 + 81\delta\sqrt{1-\phi} - 16\delta\sqrt{1-\phi}\phi)\eta^{10} + ...$$

$$(25)$$

In a similar manner, using the eq.22 in the coefficient of P^2, V_2 and θ_2 can be obtained and so forth. In this work, 4th-order solution of the problem according to eqs.17 and 18 when P=1 will be the answers as follows:

$$V(\eta) = \frac{1}{12}\eta^3 - \frac{1}{12}\eta - \frac{1}{2016}(9\delta\sqrt{1-\phi}\phi^2 + 9\delta\sqrt{1-\phi} - 18\delta\sqrt{1-\phi}\phi)\eta^7 + ...$$

$$\theta(\eta) = -0.5\eta - \frac{1}{10368}\frac{1}{(1-\phi)^{5/2}A_1}(Ec\,\Pr(\frac{1}{90}(81\delta\sqrt{1-\phi}\phi^2 + 81\delta\sqrt{1-\phi} - 16\delta\sqrt{1-\phi}\phi)\eta^{10} + ...$$

$$(26)$$

5. RESULTS AND DISCUSSION

In this study, a straightforward technique named HPM is used to solve the nonlinear equation of non-Newtonian nanofluid flow between two infinite parallel vertical flat plates. In order to show the preciseness of results, a numerical method (the fourth-order Runge–Kutta) is implemented. Tables 2, 3 and Figs. 2, 3 show the Comparisons between numerical results and HPM solutions for different parameters (δ, φ, Ec) of Cu-Water nanofluid. As it can be seen, the difference between numerical result and HPM solution is negligible indicating that the HPM method can be a trusty method for solving some such equations.

Table 2. Velocity value of Cu-water nanofluid

η	Present	Numeric
-0.75	0.05706212668	0.0563789031518292
-0.5	0.07043266381	0.0709522078393007
-0.25	0.05133340706	0.0520967264690272
0	0.01535910100	0.015700101
0.09689373164	0	0
-0.25	-0.0229822888	-0.0221489862588
0.5	-0.0487749615	-047108779075
0.75	-0.0453663958	-0.0453324291552

Table 3. Temperature value of Cu-water nanofluid

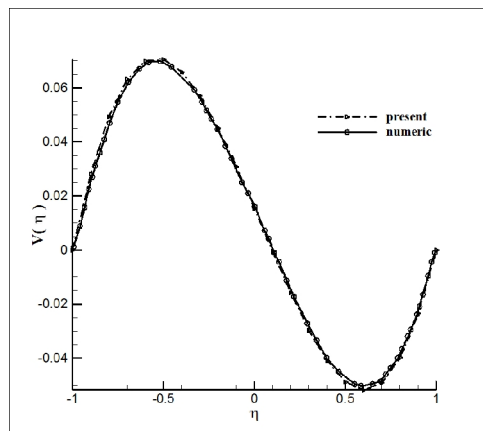| η	Present	Numeric
-0.75	0.4222082998	0.4204839230778
-0.5	0.3015480135	0.3002737376344
-0.25	0.1794516688	0.179662947314
0	0.05081001960	0.05002819772445
-0.25	-0.0877590408	-0.0880385164533
0.5	-0.2331537377	-0.2301061271274
0.75	-0.3801047403	-0.3801610899417

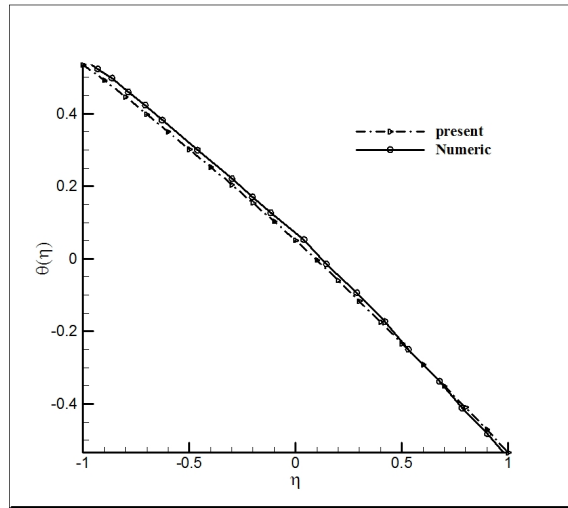**Fig. 2. Velocity Profile of Cu-Water nanofluid, Comparison between Numeric and HPM results**

Fig. 3. Temperature Profile of Cu-Water nanofluid, Comparison between Numeric and HPM results

Velocity and temperature profiles of different nanofluids are shown in Figs. 4 and 5, respectively.
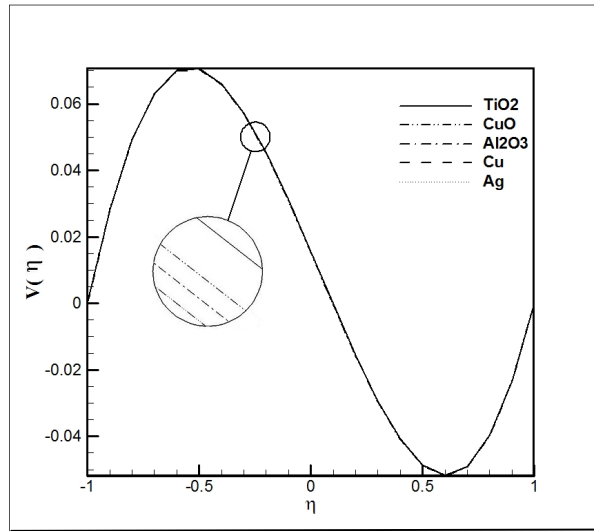

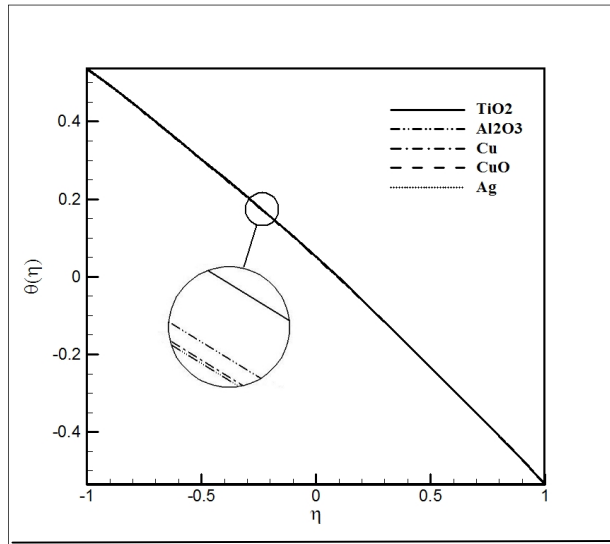**Fig. 4. Velocity profile, comparison between different nanofluids**

Fig. 5. Temperature profile, comparison between different nanofluids

All nanofluids approximately have the same trend in the velocity and temperature distributions despite the different values of their properties. Figs. 6 and 7 shows the effect of nanoparticle volume fraction (φ) on the velocity distribution $V(\eta)$ and the temperature profiles θ (η) when δ=1, Ec=1 and Pr=6.2. The volume fraction of the nanoparticles increases with velocity increase.
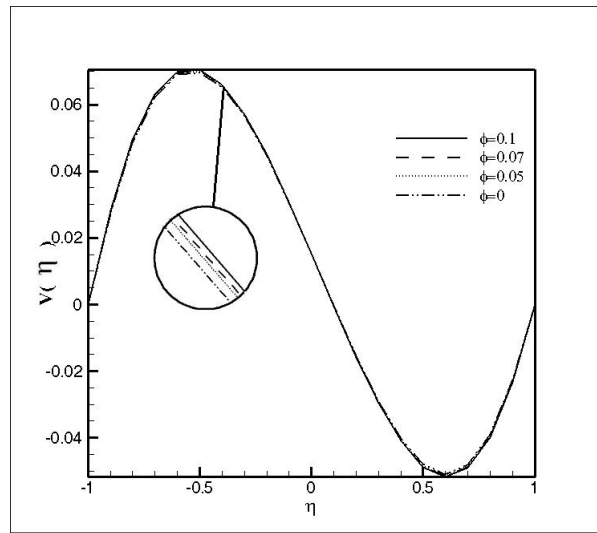
Fig. 6. Velocity profile of cu-water nanofluid, nanoparticle fraction effect
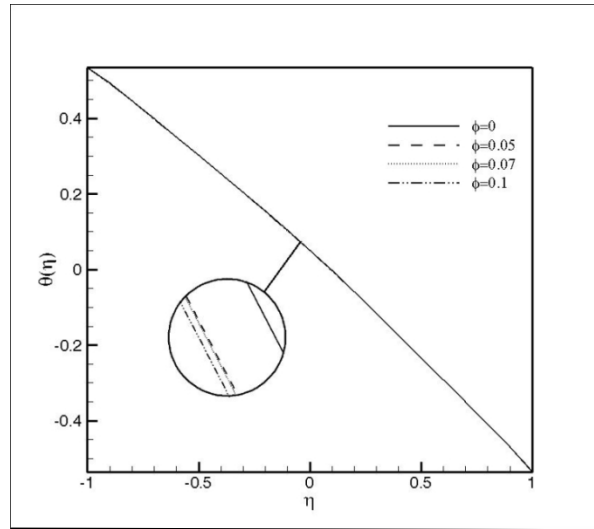
Fig. 7. Temperature profile of cu-water nanofluid, nanoparticle fraction effect

The effect of dimensionless non-Newtonian viscosity (δ) of Cu-Water nanofluid on the velocity V (η) and temperature profiles θ (η) of Cu-Water nanofluid are shown in Figs. 8 and 9 assuming φ=0.1, Ec=1 and Pr=6.2. The dimensionless non-Newtonian viscosity signifies the relative importance of the inertia effect compared to the viscous effect. Dimensionless non-Newtonian viscosity (δ) increases as velocity and temperature magnitude decrease. Also, the maximum value of velocity can be approximately observed at =0.6.
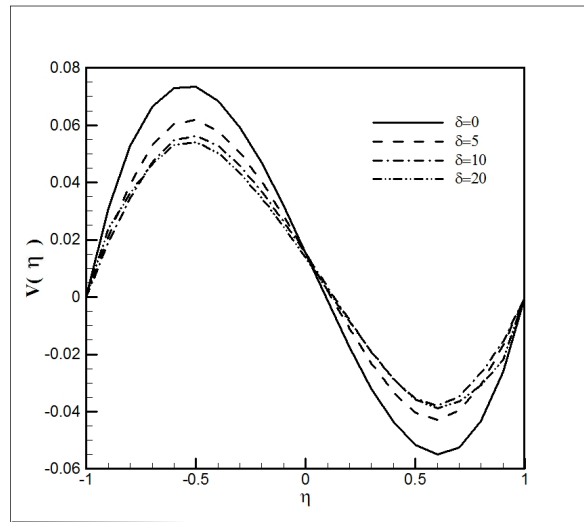
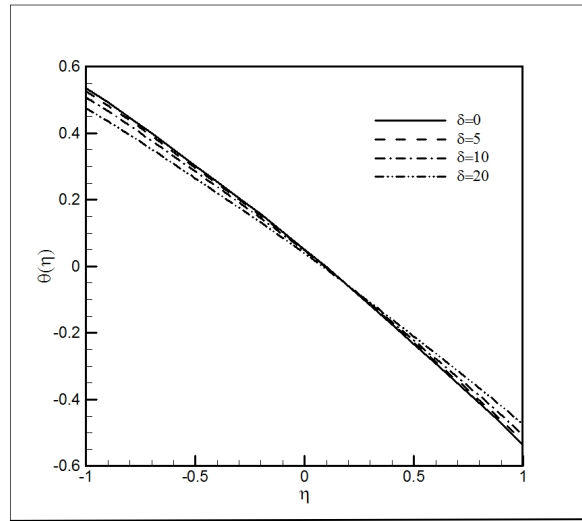**Fig. 8. Velocity profile of cu-water nanofluid, dimensionless non-newtonian viscosity effect**

Fig. 9. Temperature profile of cu-water nanofluid, dimensionless non-newtonian viscosity effect

Figs. 10 and 11 indicate the effects of the Eckert number (Ec) on the velocity V (η) and temperature distributions θ (η) of Cu-Water nanofluid when φ=0.1, δ=1 and Pr=6.2, respectively. As can be seen, increasing in the velocity and temperature will lead to an increase in Eckert number. The minimum values for the velocity and temperature will be gotten by neglecting the viscous dissipation. Also, the maximum value of the velocity can be approximately seen at │η│=0.5 in the middle of the distance between two plates, and the maximum value of the temperature is seen when │η│=1.
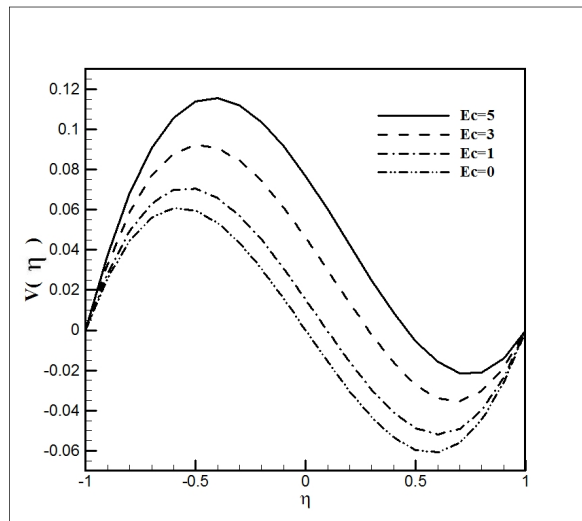

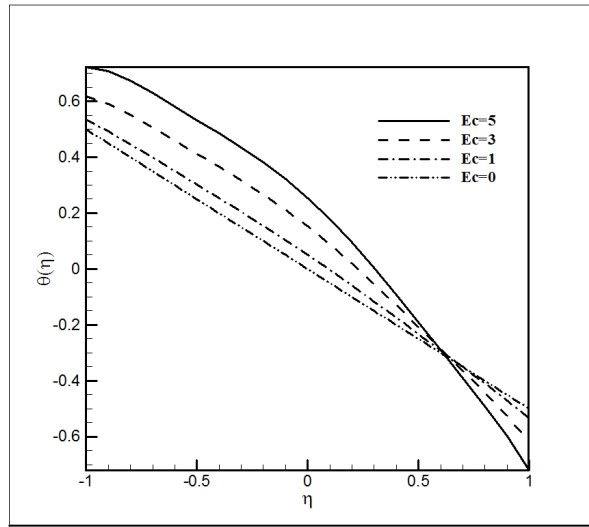**Fig. 10. Velocity profile of cu-water nanofluid, eckert number effect**

Fig. 11. Temperature profile of Cu-water nanofluid, eckert number effect

Fig. 12 shows the effect of different nanoparticle volume fraction (φ) of Cu-Water nanofluid on the skin friction (s) while the dimensionless non-Newtonian viscosity (δ) varies between 0 to 10 and the Eckert number (Ec) value is1. The skin friction has a decreasing trend with the dimensionless non-Newtonian viscosity parameter and increasing trend with the volume fraction of the nanoparticles (when $0<\delta<5$) but this trend has an opposite form (when $5<\delta<10$).
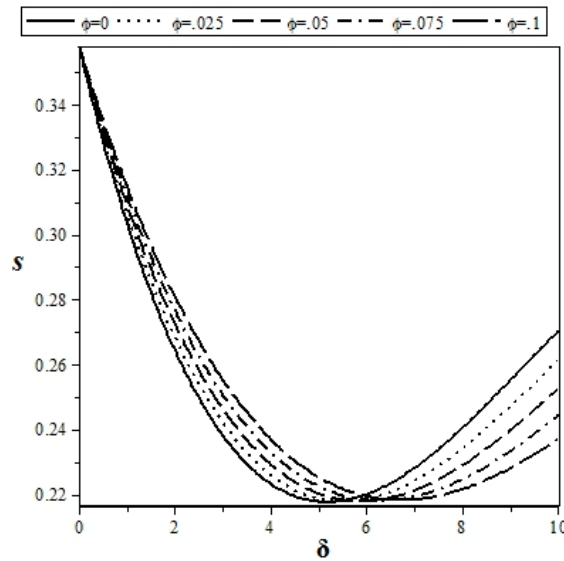

Fig. 12. Variation of skin friction, nanoparticles fraction effect

Fig. 13 shows the effect of different nanoparticle volume fraction (φ) of Cu-Water nanofluid on the heat transfer coefficient (h) while the dimensionless non-Newtonian viscosity (δ) varies among 0 to 20 and the Eckert number (Ec) value is1. The heat transfer coefficient (h)

has an increasing trend with the dimensionless non-Newtonian viscosity parameter and the volume fraction of the nanoparticles when $0<\delta<12$, but the heat transfer coefficient (h) decreases with increase of dimensionless non-Newtonian viscosity when $12<\delta<20$.
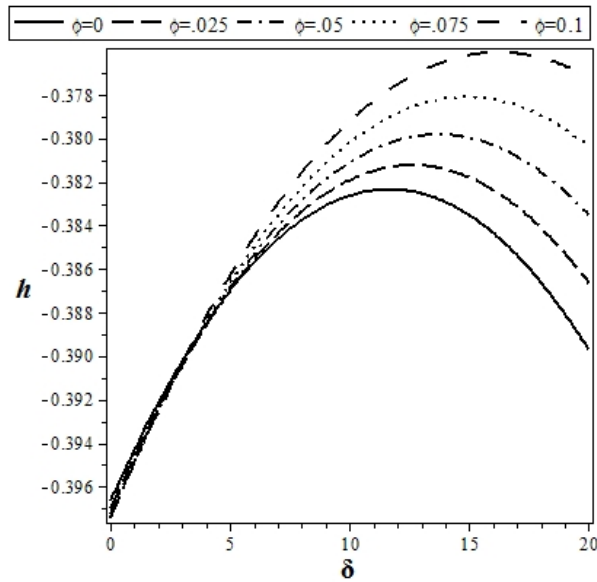

Fig. 13. Variation of heat transfer coefficient, nanoparticle fraction effect

6. CONCLUSION

In this study, the phenomenon of the free convective heat transfer of nanofluid to the power-law non-Newtonian flow between two infinite parallel vertical flat plates has been assessed by Homotopy perturbation Method. The effects of the nanoparticle volume fraction (φ), dimensionless non-Newtonian viscosity (δ) and Eckert number (Ec) on the velocity and temperature profiles have been investigated for a Cu-Water nanofluid. Finally, for different values of dimensionless non-Newtonian viscosity (δ), the effects of different value of nanofluid nanoparticle volume fraction (φ) on the Heat transfer coefficient (h) and skin friction (S) were presented and discussed. The study appears to show that an increase in δ when E and *Pr* are constant decrease the skin friction but increase the heat transfer.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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