Journal of Intelligent Learning Systems and Applications, 2015, 7, 11-20
Published Online February 2015 in SciRes. http://www.scirp.org/journal/jilsa
http://dx.doi.org/10.4236/jilsa.2015.71002

Experimental Design and Its Posterior Efficiency for the Calibration of Wearable Sensors

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Received 21 January 2015; accepted 8 February 2015; published 11 February 2015

Abstract

This paper investigates experimental design (DoE) for the calibration of the triaxial accelerometers embedded in a wearable micro Inertial Measurement Unit (μ-IMU). Firstly, a new linearization strategy is proposed for the accelerometer model associated with the so-called autocalibration scheme. Then, an effective Icosahedron design is developed, which can achieve both *D*-optimality and *G*-optimality for linearized accelerometer model in ideal experimental settings. However, due to various technical limitations, it is often infeasible for the users of wearable sensors to fully implement the proposed experimental scheme. To assess the efficiency of each individual experiment, an index is given in terms of desired experimental characteristic. The proposed experimental scheme has been applied for the autocalibration of a newly developed μ-IMU.

Keywords

Wearable Health Monitoring, IMU, Triaxial Accelerometer, Autocalibration, DoE, Modelling, Linerization

1. Introduction

Wearable health monitoring system is one of the most promising technologies to provide effective solutions to health monitoring for aging populations. Various wearable sensors equipped with artificial intelligence, e.g., neural networks, fuzzy logical, genetic algorithm, particle swarm optimization, and clustering, have already been utilized for specific health monitoring tasks [1]. However, in general, the accuracy of the wearable sensors needs to be substantially enhanced in order to improve the reliability of wearable systems to meet medical device standards.

How to cite this paper: Ye, L. and Su, S.W. (2015) Experimental Design and Its Posterior Efficiency for the Calibration of Wearable Sensors. *Journal of Intelligent Learning Systems and Applications*, **7**, 11-20.
http://dx.doi.org/10.4236/jilsa.2015.71002

With the rapid development of Micro-Electro-Mechanical Systems (MEMS) technology, chip-based wearable sensors are becoming small, inexpensive, lightweight, and low energy-consuming, which stimulate their applications in the development of wearable systems in health monitoring [2], e.g., gait analysis and fall detection/ prediction [3]. However, due to their fabrication process, similar to most wearable sensors, MEMS sensors have large bias instability and output noise. Regular calibrations are therefore necessary to improve the accuracy of sensors' measurements. However, due to the inaccessible of laboratory equipments, users of wearable health monitoring systems are normally unable to implement designed experiment sufficiently.

Several recent papers [4]-[6] report that a new calibration method for MEMS triaxial accelerometer, recognized as autocalibration, can be implemented in non-experimental condition. However, the quality, especially the assessment of each individual calibration, has not received the attention it deserves. To authors' best knowledge, unlike traditional calibration method, there is no paper systemically discussing the issues of Experimental Design (DoE) yet. Most studies concentrate on the parameter estimation algorithms and its feasibility investigations. Few papers [7] [8] qualitatively described the selections of experimental observations.

This paper aims to provide a systematic investigation of Experimental Design (DoE) for autocalibration method. A major focus of DoE is to optimally design suitable input signals to stimulate the system significantly so that the information about the system can be extracted from the experiments. For the identification of a static model of an inertial sensor, a well selected/designed set of experimental observations with desired properties, in terms of DoE, can significantly improve the accuracy of parameter estimation [9].

Classical accelerometer calibration, normally carried out in a well-controlled laboratory environment, can be formulated as a static linear parameter identification problem, for which DoE theory has been well established [9]-[13]. However, the models associated with the autocalibration are often nonlinear. This makes the linear DoE approaches, which are effective and theoretically rigorous in traditional accelerometer calibration, invalid for autocalibration.

In this study, a new linearization method for autocalibration [4] is developed in order to utilize linear DoE for this new calibration method. A 12-observation Icosahedron design has been proposed for a linearized 9-parameter model [6] to improve the accuracy of autocalibration. Two performance indices of optimal experimental design, *D*-optimality and *G*-optimality, are investigated based on the analysis of the information matrix of this Icosahedron design. Furthermore, a posterior type *D*-efficiency [11] is introduced to evaluate a specific experiment when compared with the *D*-optimal value under ideal experimental conditions.

The paper is structured as follows. The next section introduces the proposed linearization method for the 9-parameter model for autocalibration. In Section 3, an experimental design is proposed and the details of its indices will be analysed. Section 4 shows experimental validation of the designed experiment and Section 5 concludes the paper.

2. New Linearization Method for Auto-Calibration Model

A classical static linear second-order model for an accelerometer can be written as follows:

$$y=\beta_0+\sum_{i=1}^{3}\beta_i x_i+\sum_{i<j}\sum\beta_{ij}x_i x_j+\sum_{i=1}^{3}\beta_{ii}x_i^2+\varepsilon, \tag{1}$$

where x_1, x_2, x_3 are the associated control or input variables (*i.e.*, the input acceleration for each axis), $\beta_i\left(\beta_{ii}\right)$ are the unknown constant coefficients (also referred to as parameters), and ε represent the random errors in experimental measurements.

Assume a set of N experimental observations have been performed. Then, the matrix form of the experiment can be expressed as follows [9]:

$$Y=XB+\varepsilon, \tag{2}$$

where $B\in R^p$ is the vector of the unknown parameters, $Y\in R^{N\times 1}$ is the vector of measurements, matrix $X\in R^{N\times p}$ is generated by the input signals, and $\varepsilon\in R^{N\times 1}$ is the vector of random errors.

Assume that the random errors $\varepsilon\in R^{N\times 1}$ are zero mean, then the information matrix for B in Equation (2) can be defined as $X^{\mathrm{T}}X$:

$$X^{\mathrm{T}}X=\sum_{i=1}^{N}f\left(x_i\right)f^{\mathrm{T}}\left(x_i\right), \tag{3}$$

where $f^{\mathrm{T}}(x_i)$ is the *i*-th row of matrix *X* [14]. For a specific *N* trials design ξ_N, Equation (3) can be normalized as:

$$M(\xi_N) = \frac{X^{\mathrm{T}} X}{N}, \tag{4}$$

which is also known as moment matrix. If $X^{\mathrm{T}} X$ is full rank, the variance-covariance matrix of the least square (LS) estimator $\hat{B}$ is:

$$\operatorname{var}(\hat{B}) = (X^{\mathrm{T}} X)^{-1} \sigma^2. \tag{5}$$

The variance of $\hat{y}(x)$ is of the form:

$$\operatorname{var}(\hat{y}(x)) = \sigma^2 f^{\mathrm{T}}(x)(X^{\mathrm{T}} X)^{-1} f(x). \tag{6}$$

In order to compare within different experimental designs, the scaled prediction variance is often defined as follows [14]:

$$d(x, \xi_N) = f^{\mathrm{T}}(x) M^{-1}(\xi_N) f(x) = \frac{N \operatorname{var}(\hat{y}(x))}{\sigma^2}. \tag{7}$$

To apply DoE theory for the calibration of MEMS accelerometer, we define uncalibrated acceleration generated from accelerometer output as $V = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^{\mathrm{T}}$, which is generated from the measurement of accelerometers. We also define $O = \begin{bmatrix} o_x & o_y & o_z \end{bmatrix}^{\mathrm{T}}$ as the offset of the accelerometer. The vector $A = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}^{\mathrm{T}}$ is the real acceleration component on each axis.

A model describing the accelerometer can then be expressed in matrix form as below:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \left(\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} + \begin{bmatrix} o_x \\ o_y \\ o_z \end{bmatrix} \right), \tag{8}$$

where S is the scale factor matrix. The diagonal elements S_{ii} represent sensitivity of each direction and off-diagonal elements S_{ij} represent cross-axis sensitivity. Considering the symmetry constraint for the scale factor matrix S (*i.e.*, $S_{ij} = S_{ji}$ [6]). Therefore, the model in Equation (8) can be expressed in 9 independent parameters.

The autocalibration method is based on the fact that the overall acceleration which is measured by triaxial accelerometer should equal to the local gravity acceleration "1*g*" in static condition. The principle of autocalibration is:

$$g = \sqrt{a_x^2 + a_y^2 + a_z^2}. \tag{9}$$

By applying the method of autocalibration (see Equation (9)) for the 9-parameter model from Equation (8), we have:

$$\begin{aligned} g^2 &= a_x^2 + a_y^2 + a_z^2 + \gamma \\ &= \left[S_{xx} \cdot (V_x + O_x) + S_{xy} \cdot (V_y + O_y) + S_{xz} \cdot (V_z + O_z) \right]^2 \\ &\quad + \left[S_{xy} \cdot (V_x + O_x) + S_{yy} \cdot (V_y + O_y) + S_{yz} \cdot (V_z + O_z) \right]^2 \\ &\quad + \left[S_{xz} \cdot (V_x + O_x) + S_{yz} \cdot (V_y + O_y) + S_{zz} \cdot (V_z + O_z) \right]^2 + \gamma \end{aligned} \tag{10}$$

where γ is squared difference between accelerometer output and local gravity acceleration "1*g*". Equation (10) can be further expressed as:

$$g^2 = \sum_{i=x,y,z} \left[\sum_{j=x,y,z} S_{ij} \cdot (V_j + O_j) \right]^2 + \gamma \tag{11}$$

where $S_{ij} = S_{ji}$.

Equation (11) cannot be written in the form of Equation (1) because of its nonlinearity with the parameters. To estimate the parameters, most existing studies use either nonlinear least square method [4] [11] or nonlinear recursive algorithms [7]. However, the key of these approaches is to locally linearize the nonlinear Equation (11) and recursively identify the unknown parameters.

Inspired by these studies, this paper proposes a new linearization scheme to directly linearize Equation (11) and transform it in the form of Equation (1). From this, mature linear DoE theory can be directly applied to handle experimental design and parameter estimation for the autocalibration scheme. In contrast with local linearization (e.g., Taylor expansion around the observation point), the main strategy of the proposed linearization method is based on re-combination of parameters.

Firstly, this approach disregards the items in Equation (11) which have little impact on parameter estimation. **Table 1** indicates that zero-*g* offset for each axis could be quite large in the worst case. If necessary, a pre-calibration is recommended to reduce initial zero-*g* offsets. After this procedure, the residual O_i in Equation (12) will be much smaller when comparing to local gravity acceleration "1*g*". Let us compute and simplify a_x:

$$a_x^2 = \sum_{i=x,y,z} S_{xi}^2 \left(V_i^2 + 2V_iO_i + O_i^2\right) + \sum_{i \neq j}\sum S_{xi}S_{xj}\left(V_iV_j + V_iO_j + V_jO_i + O_iO_j\right). \tag{12}$$

From **Table 1**, we can see the cross-axis sensitivity S_{ij} is only 1%. Therefore, the terms in Equation (12) contain $S_{xi(j)}O_{j(i)}$ and $S_{xi}S_{xj}$ which $i, j \neq x$ can be disregarded. We also disregard the items contained O_i^2 and O_iO_j as these items can be reduced significantly after pre-calibration. The remains of a_x^2 is:

$$a_x^2 = S_{xx}^2V_x^2 + 2S_{xx}^2V_xO_x + 2S_{xx}S_{xy}V_xV_y + 2S_{xx}S_{xz}V_xV_z. \tag{13}$$

We can apply the same simplification method for a_y and a_z to simplify Equation (11) as follows:

$$g^2 = \sum_{i=x,y,z} S_{ii}^2V_i^2 + \sum_{i=x,y,z} 2S_{ii}^2V_iO_i + \pm\sum_{i \neq j}\sum\left(S_{ii} + S_{jj}\right)S_{ij}V_iV_j + \tilde{\varepsilon} + \bar{\varepsilon}, \tag{14}$$

where $S_{ij} = S_{ji}$, $\tilde{\varepsilon}$ is zero mean Gaussian noise and $\bar{\varepsilon}$ is non-zero random error representing the mean of the summation of the disregarded items.

From Equation (14), let us define new parameters for the re-combined parameters as follows:

$$\begin{aligned} S_{xx}^2 &= \beta_{11} \\ S_{yy}^2 &= \beta_{22} \\ S_{zz}^2 &= \beta_{33} \\ 2S_{xx}^2O_x &= \beta_1 \\ 2S_{yy}^2O_y &= \beta_2 \\ 2S_{zz}^2O_z &= \beta_3 \\ 2\left(S_{xx} + S_{yy}\right)S_{xy} &= \beta_{12} \\ 2\left(S_{xx} + S_{zz}\right)S_{xz} &= \beta_{13} \\ 2\left(S_{yy} + S_{zz}\right)S_{yz} &= \beta_{23} \end{aligned} \tag{15}$$

Let us use $V_{1,2,3}$ to represent $V_{i,j,k}$, Equation (14) can be expressed as:

$$y = \beta_1V_1 + \beta_2V_2 + \beta_3V_3 + \beta_{11}V_1^2 + \beta_{22}V_2^2 + \beta_{33}V_3^2 + \beta_{12}V_1V_2 + \beta_{13}V_1V_3 + \beta_{23}V_2V_3 + \tilde{\varepsilon} + \bar{\varepsilon}. \tag{16}$$

This can be simplified as:

$$y = \sum_{i=1}^{3}\beta_iV_i + \sum_{i=1}^{3}\beta_{ii}V_i^2 + \sum_{i<j}\sum\beta_{ij}V_iV_j + \tilde{\varepsilon} + \bar{\varepsilon}. \tag{17}$$

Since V_i is input signal, the equation is now a linear equation about unknown parameters $\beta_i\left(\beta_{ii}\right)$. If we tentatively disregard $\bar{\varepsilon}$ (in Section 4, we will show this unknown number can be recursively estimated), Equation (17) becomes a special case of Equation (1).

Table 1. Some significant specifications of ADXL345.

Parameter	Min	Typ	Max	Unit
Cross-axis		±1		%
Sensitivity (2 g range)	230	256	282	LBS/g
0 g offset for *X*, *Y*	−150	0	150	mg
0 g offset for *Z*	−250	0	250	mg
Offset vs. temperature *X*, *Y*		±0.4		mg/˚C
Offset vs. temperature *Z*		±1.2		mg/˚C

Applying linear least square estimation (LSE) method for the simplified linear model, we have

$$\hat{B} = \left(X^{\mathrm{T}} X\right)^{-1} X^{\mathrm{T}} Y \tag{18}$$

where

$$X = \begin{bmatrix} V_1 & V_2 & V_3 & V_1^2 & V_2^2 & V_3^2 & V_1V_2 & V_1V_3 & V_2V_3 \end{bmatrix}$$

and

$$\hat{B} = \begin{bmatrix} \hat{\beta}_1 & \hat{\beta}_2 & \hat{\beta}_3 & \hat{\beta}_{11} & \hat{\beta}_{22} & \hat{\beta}_{33} & \hat{\beta}_{12} & \hat{\beta}_{13} & \hat{\beta}_{23} \end{bmatrix}^{\mathrm{T}}.$$

Based on linear least square method, all new unknown parameters from Equation (16) can be estimated. According to the definition of Equation (15), the original 9 independent parameters S_{ii}, S_{ij} and O_i can therefore be computed. However, as the nonzero random error ε is disregarded, to obtain desired estimation accuracy, the LSE method should be recursively performed (see Section 4 for details). It should be noticed that β_i and β_{ii} are not independent. For example, both β_1 and β_{11} include common term S_{xx}^2 from the definition above.

3. Proposed Experimental Plan and Optimality Indices

3.1. Icosahedron Design

In order to estimate 9 unknown parameters, a minimum number of 9 observations are necessary. To balance the cost and accuracy, we propose a 12-observation Icosahedron design, which is a space filling design aiming for the uniformly distribution of experimental observations on experimental domain. This experimental design is for the linearized 9-parameter model derived in Section 2. The idea of Icosahedron design is that all 12 observations uniformly distribute on the surface of sphere.

Due to the constraint of the gravity based calibration, all the experimental observations will be situated uniformly on the surface of a sphere whose radius equals to local gravity "1*g*". In another word, these 12 experimental observations will construct an Icosahedron whose circumcircle has radius of "1*g*".

For Icosahedron design, if the radius of its circumcircle is "1*g*", then all 12 observations can be pinpointed on rectangular coordinate system (see Table 2 for details). In Table 2, $a = \sqrt{\frac{2}{5+\sqrt{5}}}$, $b = \frac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}}$.

For each individual observation, the relationship of Table 2 can be well described by the following equations:

- $A = \cos\theta_x$, A is tilt angle between x and gravity;
- $B = \cos\theta_y$, B is tilt angle between y and gravity;
- $C = \cos\theta_z$, C is tilt angle between z and gravity.

3.2. *G*-Optimality

G-optimal design is seeking to minimize the maximum value of the scaled prediction variance in Equation (7) over the experimental region [14]:

$$\min_{\xi} \left[\max_{x \in R} \left\{ d\left(x, \xi_N\right) \right\} \right]. \tag{19}$$

Table 2. Three factors icosahedron design for triaxial accelerometer model and the tilt angle in three dimensional coordinate.

Observation	*A*		*B*		*C*	
1	0	(90°)	$-a$	(148.3°)	$-b$	(121.7°)
2	0	(90°)	a	(31.7°)	$-b$	(121.7°)
3	0	(90°)	$-a$	(148.3°)	b	(58.3°)
4	0	(90°)	a	(31.7°)	b	(58.3°)
5	$-a$	(148.3°)	$-b$	(121.7°)	0	(90°)
6	a	(31.7°)	$-b$	(121.7°)	0	(90°)
7	$-a$	(148.3°)	b	(58.3°)	0	(90°)
8	a	(31.7°)	b	(58.3°)	0	(90°)
9	$-b$	(121.7°)	0	(90°)	$-a$	(148.3°)
10	b	(58.3°)	0	(90°)	$-a$	(148.3°)
11	$-b$	(121.7°)	0	(90°)	a	(31.7°)
12	b	(58.3°)	0	(90°)	a	(31.7°)

G-optimal is an important measurement of performance which indicates satisfactory prediction of output throughout the design region.

We propose the following theorem to show the proposed Icosahedron design is *G*-optimal.

Theorem 1. The proposed Icosahedron design for the linearized 9-parameter accelerometer model

$$y=\sum_{i=1}^{3}\beta_i x_i+\sum_{i=1}^{3}\beta_{ii}x_i^2+\sum_{i<j}\sum\beta_{ij}x_i x_j+\tilde{\varepsilon} \tag{20}$$

is *G*-optimal.

Proof. The variance equation of predicted $\hat{y}(x)$ is:

$$\operatorname{var}\left(\hat{y}(x)\right)=\sigma^2 f^{\mathrm{T}}(x)\left(X^{\mathrm{T}}X\right)^{-1}f(x). \tag{21}$$

Recall scaled prediction variance from Equation (7):

$$d\left(x,\xi_N\right)=f^{\mathrm{T}}(x)M^{-1}\left(\xi_N\right)f(x)=\frac{N\operatorname{var}\left(\hat{y}(x)\right)}{\sigma^2}.$$

A *G*-optimal design ξ is one which can min-max $d(x,\xi)$, *i.e.*,

$$\min_{\xi}\left[\max_{x\in R}d(x,\xi)\right]. \tag{22}$$

Regarding Equation (21), *G*-optimal is equivalent to

$$\min_{\xi}\left[\max_{x\in R}\left\{f^{\mathrm{T}}(x)M^{-1}\left(\xi_N\right)f(x)\right\}\right] \tag{23}$$

where M is the moment matrix $X^{\mathrm{T}}X/N$.

According to [9]

$$\max_{x\in R}\left\{d\left(x,\xi_N\right)\right\}\geq p \tag{24}$$

where p is the number of parameters.

That is, for a specific experimental design, if

$$\max_{x\in R}\left\{d\left(x,\xi_N\right)\right\}=p \tag{25}$$

then this experimental design is G-optimal design [9].

Consider the Icosahedron design proposed above for 9-parameter model:

$$y=\sum_{i=1}^{3}\beta_i x_i+\sum_{i=1}^{3}\beta_{ii}x_i^2+\sum_{i<j}\sum\beta_{ij}x_i x_j+\tilde{\varepsilon}.$$

Recall Icosahedron design, matrix X in Equation (21) is:

$$X=\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_1^2 & x_2^2 & x_3^2 & x_1x_2 & x_1x_3 & x_2x_3 \end{matrix} \\ \begin{bmatrix} 0 & -a & -b & 0 & a^2 & b^2 & 0 & 0 & ab \\ 0 & a & -b & 0 & a^2 & b^2 & 0 & 0 & -ab \\ 0 & -a & b & 0 & a^2 & b^2 & 0 & 0 & -ab \\ 0 & a & b & 0 & a^2 & b^2 & 0 & 0 & ab \\ -a & -b & 0 & a^2 & b^2 & 0 & ab & 0 & 0 \\ a & -b & 0 & a^2 & b^2 & 0 & -ab & 0 & 0 \\ -a & b & 0 & a^2 & b^2 & 0 & -ab & 0 & 0 \\ a & b & 0 & a^2 & b^2 & 0 & ab & 0 & 0 \\ -b & 0 & -a & b^2 & 0 & a^2 & 0 & ab & 0 \\ b & 0 & -a & b^2 & 0 & a^2 & 0 & -ab & 0 \\ -b & 0 & a & b^2 & 0 & a^2 & 0 & -ab & 0 \\ b & 0 & a & b^2 & 0 & a^2 & 0 & ab & 0 \end{bmatrix} \end{array}.$$

Substituting the value of $a=\sqrt{\frac{2}{5+\sqrt{5}}}$ and $b=\frac{1+\sqrt{5}}{\sqrt{10+2\sqrt{5}}}$ into Icosahedron design, as a result, we can compute the inverse of the moment matrix $M^{-1}(\xi_N)$:

$$M^{-1}(\xi_N)=\begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & -1.5 & -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & 6 & -1.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & -1.5 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix}.$$

Considering that

$$f(x)=\begin{bmatrix} x_1 & x_2 & x_3 & x_1^2 & x_2^2 & x_3^2 & x_1x_2 & x_1x_3 & x_2x_3 \end{bmatrix}^T,$$

we have

$$\begin{aligned} d(x,\xi_N) &= f^{\mathrm{T}}(x)M^{-1}(\xi_N)f(x) \\ &= 3\left(x_1^2+x_2^2+x_3^2\right)+6\left(x_1^2+x_2^2+x_3^2\right)^2. \end{aligned} \tag{26}$$

Under the constrain $x_1^2+x_2^2+x_3^2=1$, it is easy to see:

$$\max_{x\in R}\{d(x,\xi_N)\}=9.$$

According to the theorem of G-optimal in [4], this 12-observation Icosahedron design for the linearized 9-parameter model is G-optimal. □

3.3. *D*-Optimality

Another desired design characteristic of experimental design is *D*-optimality. The criterion of *D*-optimality is maximizing the determinant of the information matrix for continuous design or moment matrix for exact design [14]:

$$D = \max_{\xi} \det\left(M\left(\xi\right)\right), \tag{27}$$

which leads to minimize the size of the confidence ellipsoid for the estimator $\hat{B}$ in Equation (5).

Kiefer and Wolfowitz [15] [16] developed the well-known Equivalence Theorem (KWT theorem), which provides a practical way to check if a design is *D*-optimal. This theorem shows that for continuous designs, *D*- and *G*-optimal designs are equivalent under some standard assumptions [15] [16].

Based on KWT theorem, we show the proposed Icosahedron design is also *D*-optimal.

Theorem 2. *The proposed Icosahedron design is D-optimal for the linearized* 9*-parameter model*

$$y = \sum_{i=1}^{3}\beta_i x_i + \sum_{i=1}^{3}\beta_{ii} x_i^2 + \sum_{i<j}\sum \beta_{ij} x_i x_j + \tilde{\varepsilon}.$$

Proof. Let us consider a continuous experimental design ξ with finite observation *N*. From Equation (21), if $\max_{x\in R} d\left(x,\xi\right) = p$, then this experiment is continuous *G*-optimal design. For our Icosahedron design, we proved that $\max_{x\in R} d\left(x,\xi_N\right) = 9$ in Theorem 1.

To convert exact design ξ_N to continuous design ξ, let us consider that each observation in the experimental design shares the same weight. In this case, the value of $\max_{x\in R} d\left(x,\xi\right)$ for our Icosahedron design will also be 9 which means Icosahedron design is continuous *G*-optimal design for the linearized 9-parameter model.

Based on KWT Equivalence Theorem [14]-[16], since the Icosahedron design can be considered as continuous *G*-optimal design, it is also continuous *D*-optimal design. □

4. Experimental Results and Discussion

The μ-IMU that we use to test the Icosahedron design contains a triaxial accelerometer ADXL345 manufactured by Analog Device. The main characteristics of ADXL345 are listed in **Table 1**.

Based on the described experimental design in Section 3, we tried to implement the Icosahedron design. It is not supervising that the proposed plan cannot be fully implemented by using the "non-professional" calibration devices. To access the quality of a specific experiment, we adopt the following index, *D*-efficiency [10], to evaluate the quality of a particular experiment

$$D_{\text{eff}} = \left(M\left(\xi_N\right)/M\left(\xi_N^*\right)\right)^{1/P}, \tag{28}$$

where ξ_N^* and ξ_N stand for the designed optimal experiment and a specific experiment respectively.

To evaluate the efficiency of a specific experiment, ideally, we need the exact input value of each observation for a specific design ξ_N. In a traditional accelerometer calibration experiment, the input (acceleration) to the accelerometer can be accurately measured and its value are adjustable. Therefore, for traditional accelerometer calibration, the *D*-efficiency can be determined even before the experiments.

However, for auto-calibration, the input value on each axis of a particular observation, denoted by a vector *A*, cannot be directly measured from the calibration device. We therefore have to use the output of the accelerometer, which is under calibration, to estimate the real input acceleration *A*. Equation (8) describes the relationship between the uncalibrated acceleration output *V* and the real acceleration input *A* if assuming the scale factor and offset are accurate. Let us recall Equation (8) and simplify it as:

$$A = S\left(V + O\right). \tag{29}$$

In order to obtain real acceleration *A*, we need to compute scale factor *S* and offset *O* first. Towards the end of Section 2, we mentioned the scale factor S and offset O can be recursively estimated by LSE. Based on Equation (17), the linearized 9-parameter triaxial accelerometer model can be rewrited as:

$$Y = V_1 B_1 + \tilde{\varepsilon}_1 + \overline{\varepsilon}, \tag{30}$$

where *Y* is local gravity "1 g", V_1 represents uncalibrated acceleration from accelerometer, B_1 is vector of re-combined parameters defined in Equation (15), $\tilde{\varepsilon}$ is zero mean error and $\bar{\varepsilon}$ is non zero error representing the mean of disregarded items. To apply LSE for Equation (30), let us disregard $\bar{\varepsilon}$:

$$Y = V_1 B_1 + \tilde{\varepsilon}_1, \tag{31}$$

where Y is local gravity "1 g", V_1 is the known quantity from accelerometer and B_1 is re-combined parameter by scale factor S and offset O. S_1 and O_1 can then be solved by using LSE as shown in Equation (18).

From Equation (29), we have

$$A_1 = S_1 \left(V_1 + O_1 \right). \tag{32}$$

Due to the fact that we neglected some little impact items during LSE, A_1 will not be exactly the same as real acceleration A, but A_1 is closer to real acceleration A comparing to V_1. In this case, when A_1 is closer to A, the value of offset O will be reduced. Recall from Section 2 that all disregarded items contain offset O, it means the mean of the summation of all disregarded items $(\bar{\varepsilon})$ will reduce. We replace V_1 with A_1 (A_1 is marked as V_2 in Equation (33) and apply LSE again for the equation below with less impact from $\bar{\varepsilon}$:

$$Y = V_2 B_2 + \tilde{\varepsilon}_2. \tag{33}$$

From Equation (15) and Equation (18), the new scale factor S_2 and offset O_2 can then be solved. Recall Equation (29):

$$A_2 = S_2 \left(V_2 + O_2 \right). \tag{34}$$

In this case, A_2 will be even closer to real acceleration A comparing to $A_1 \left(V_2 \right)$.

Let us repeat this procedure, A_i is approaching to real acceleration A while offset O is reducing to 0. The accuracy of LSE will increase because all disregarded items contain offset O will drop to 0. Eventually, offset O and cross-axis factors S_{ij_n} will be 0, sensitivity factor of each direction S_{ii_n} will be 1. Then this acceleration A_n will be optimal estimation of real acceleration A.

The overall equation of Equation (29) is:

$$A_n = S_n \left(\cdots S_2 \left(S_1 \left(V_1 + O_1 \right) + O_2 \right) \cdots + O_n \right). \tag{35}$$

Now, as A_n can be applied for experiment evaluation, this recursive procedure can guarantee the accuracy of the evaluation for posterior type *D*-efficiency.

We performed the calibration experiment in the Center of Health Technologies (CHT), University of Technology, Sydney (UTS), without using a turntable. In contrast with the ideal setting, the posterior type *D*-efficiency for our experiment is around 99.7% which is slighter smaller than 100%. It indicates our experiment achieved desired results.

Experimental results also showed the Mean Square Error (MSE) has been reduced from 0.00344 g^2 to 0.000255 g^2 by the proposed experimental design/calibration method.

5. Conclusion

This study investigates the DoE for autocalibration of triaxial accelerometer in a wearable micro Inertial Measurement Unit (μ-IMU), and our contribution is two-fold. Firstly, a new model linearization strategy is proposed to linearize the nonlinear model associated with the autocalibration of triaxial accelerometer. The major technique of the proposed linearization strategy is based on recombination of parameters rather than local linearization around observation point (e.g. Taylor expansion). With such a linearized model, the classical linear model identification and DoE approaches can be applied to calibrate the triaxial accelerometer in a non-experimental environment. The second contribution is that this paper introduces a new experimental scheme, Icosahedron design. We have proved that this scheme is both *G*-optimal and *D*-optimal for the linearized 9-parameter triaxial accelerometer model. Experimental results also demonstrate that the proposed DoE scheme can significantly decrease the MSE of triaxial accelerometer after calibration. This indicates that the proposed linearization method is reliable and efficient. We believe that the proposed experimental design approach can provide an efficient tool for the users of wearable sensors to efficiently calibrate the sensors in free living condition.

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