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Stability and Controllability of Nonlinear Systems

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper is aimed at establishing new stability and controllability results for nonlinear systems. The approach is to use the Lyapunov indirect method to obtain the stability of the equilibrium solution of the uncontrolled nonlinear system by applying the Jacobi's linearization method and the controllability of the controlled system obtained by the rank criterion for properness. Example is given with a real-life application to illustrate the effectiveness of the theoretical results.

Keywords: Stability; Lyapunov indirect method; controllability; mass-spring-damper; non-linear system.

Subject classification: 93D05; 93B05; 93-XX.

1 Introduction

The study of nonlinear systems with control inputs has attracted lots of attention in recent years see for example [1,2,3], etc., because of its wide area of application. Although the study of nonlinear systems have made tremendous progress in recent year, there are lots of new challenges and problems existing in many areas including stability and control of such systems as seen in Xie et al. [4].

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A nonlinear control system can be defined literally as a system in which the interdependence between the control inputs and output variables have no linear combination; this implies some extra interactions between the control variables may be required even if some level of dependence can be established as linear combination from the input control variables.

For a given nonlinear control system stability is one of the most important characteristics to be investigated. There are several analysis technique for investigating the stability of nonlinear system with control inputs which includes the Lyapunov based, fixed point based and spectral radius see [5] and references therein for detailed explanation on these methods. The focus on this paper is on the Lyapunov based research; this method of research first presented by the Russian Mathematician Lyapunov in 1892 ([6]) has two methods (the direct and indirect methods of Lyapunov) for stability analysis of dynamical systems. Though the Lyapunov direct method can be used to design and analyze nonlinear systems and achieve global results on stability. Our interest is on the latter type; where instead of looking for a Lyapunov function to be applied directly to the nonlinear system, we use the idea of system linearization around a given point to achieve stability within some region (see [7]) using quadratic Lyapunov functions, and the result applied to controllability analysis of the system.

The stability of nonlinear systems has been studied by several authors including [1,8,9,10,11] and [12]. For example, in [10]; Li et al. [10] studied nonlinear second order model and equilibrium point characteristic analysis of DC traction power supply establishing results in electromagnetic transient process of traction power supply system using eigen-value analysis method. In [11]; Okumus and Soykan [11] investigated the nature of the solutions of second order nonlinear difference equations establishing results on local stability of the equilibrium point and oscillatory behaviour of the systems using Linearization and Jacobian matrix method where they ensured that none of the eigen-values has modulus greater than one.

The study of nonlinear systems have been extended to the controllability of nonlinear systems see [2,3,13] etc. In [13]; Davies and Oliver [13] studied the Null controllability of neutral systems with infinite delays by developing sufficient conditions when the values of the control lie in an m-dimensional unit cube, their conditions guarantee that, if the uncontrolled system is uniformly asymptotically stable and the control system satisfies a full rank condition then the control system is null controllable with constraint. In [3]; Klamka et al. [3] investigated the controllability of second order dynamical systems by validating results on different approaches to the problem of controllability using moment and fixed-point methods.

Motivated by the works in ([3,10,11]) we extends the methods in [3] and [11] to establish results on stability of nonlinear systems using the Lyapunov indirect method and controllability of the system by the rank criterion for properness. These results are then applied to a physical system (mass spring damper) with their simulation output results given.

The paper is organized in the following order; Section 1, contains general overview of the study background as introduction. In Section 2, preliminaries and definitions on the subject areas are given as guide to the research methodology. Section 3 contains stability results on the equilibrium point for the system while Section 4 contains the main results of this research; with application and simulation output results illustrating the effectiveness of the study given in Section 5 prior to the discussion and conclusion in Section 6.

2 Preliminaries and Definitions

Let $R = (-\infty, \infty)$, R^n is a real n – dimensional Euclidean space with norm $|\cdot|$. We consider the autonomous system

$$\dot{x} = f(x, u), \tag{2.1}$$

where $x \in R^n$, $u \in R^m$ and $f: R^n \times R^m \to R^n$ is a continuously differentiable function and define

$$f(x, u) \overset{\text{def}}{=} \bar{f}(x, 0) + Ax + Bu. \tag{2.2}$$

Here, A, B are $n \times n$ and $n \times m$ constant matrices respectively, and $A = \frac{\partial f}{\partial x}\Big|_{x=0}$, $B = \frac{\partial f}{\partial u}\Big|_{u=0}$ where $\frac{\partial f}{\partial x}$ denotes the Jaciobian matrix.

Consider system (2.1) with all its necessary assumptions given by

$$\dot{x} = Ax + \bar{f}(x,0) + Bu, \tag{2.3}$$

and its free system

$$\dot{x} = Ax, \tag{2.4}$$

Here, the variation of parameter of the system (2.3) with its initial complete state $\emptyset(t_0)$ and all its necessary assumptions following the methods in ([2], [13]) for $t = t_1$ is given by

$$x(t_1, u) = X(t_1, t_0)\emptyset(t_0) + \int_{t_0}^{t_1} X(t_1, s)\bar{f}\left(x(s)\right)ds + \int_{t_0}^{t_1} X(t_1, s)\, Bu(s)ds \tag{2.5}$$

where $X(t, s)$ is an $n \times n$ matrix function with

$\frac{\partial}{\partial t}X(t, s) = AX(., s),\;\; t \geq s$, a.e in t and in s, and

$$X(t, s)\begin{cases} 0 & t < s < 0, \\ I & t = s = 0. \end{cases}$$

Set, $Z(t_1, u) = X(t, s)B,\;\; t \geq s \geq 0$ and define the reachable set $R(t_1, t_0)$ of (2.3) as

$$R(t_1, t_0) = \int_{t_0}^{t_1} Z(t_1, s)\mathrm{u}(s)\mathrm{ds}.$$

The controllability matrix of (2.3) is given by

$$W = \int_{t_0}^{t_1} Z(t_1, s)Z^T(t_1, s)\mathrm{ds},$$

where Z^T is the transpose of Z. The admissible controls in this paper is a measurable $m-$vector valued function with $u(t)$ constrained to lie in an $m-$dimensional unit cube C^m ; where, $C^m = \{u \in R^m\colon |u_j| \leq$ *1,j=1,2, ...,m.*

We now give some definitions that underpins the subject areas of this research work.

Definition 2.1. The equilibrium point $x = 0$ of system (2.3) is stable if for any $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$ such that if $\| x(0) \| < \delta \Longrightarrow \| x(t) \| < \varepsilon$, for all $t \geq 0$. The origin is stable if any trajectory starting from inside the ball $B(0, \delta) = \{x \in R^n\colon \|x\| < \delta\}$ remains within the $B(0, \epsilon) = \{x \in R^n\colon \|x\| < \varepsilon\}$, otherwise the equilibrium point is unstable.

Definition 2.2. The equilibrium point $x = 0$ of system (2.3) is asymptotically stable if for any $\varepsilon > 0$ there exists a $\delta = \delta(\varepsilon) > 0$ such that if $\| x(0) \| < \delta \Longrightarrow \lim_{t\to\infty} x\,(t) = 0$. In this case the ball $B(0, \delta) =$

$\{x \in R^n: \|x\| < \delta\}$ is called the attraction domain of the equilibrium point. Therefore, the attraction domain is the set of all initial points that the equilibrium point is asymptotically stable.

Definition 2.3. System (2.3) is Euclidean null controllable with constraints on $[t_0, t_1]$ if for each $\emptyset \in R^n$, there exists a $t_1 > t_0$ and $u \in C^m$ such that the solution $x(t,u)$ of (2.3) with $u = 0$ satisfies $x(.,u) = \emptyset$ and $x(t,u) = 0 = t_1 > t_0$.

3 Stability Result

Consider system (2.1) with $u = 0$ given by

$$\dot{x} = f(x, 0) \tag{3.1}$$

Theorem 3.1. Lyapunov's indirect method

If $x = 0$ is an equilibrium point for the system (3.1) with $f(0,0) = 0$, for all $t \geq 0$. Let

$$A = \left.\frac{\partial f(x,0)}{\partial x}\right|_{x=0} \tag{3.2}$$

be the Jacobian matrix of f with respect to x at the origin such that

$$\bar{f}(x,0) = f(x,0) - Ax \tag{3.3}$$

and assume that

$$\lim_{\|x\|\to 0} \frac{\|\bar{f}(x,\ 0)\|}{\|x\|} = 0 \tag{3.4}$$

Furthermore, let A be defined by equation (3.2), so that it can be approximated by

$$\dot{x} = Ax \tag{3.5}$$

Then, the origin is

a. Asymptotically stable if the origin of the linearized system (3.5) is asymptotically stable, i.e. if the matrix A is Hurwitz namely the eigenvalues of A lies on $\mathbb{C}^-$
b. Unstable if the origin of the linearized system (3.5) is unstable i.e. if one or more eigenvalues of A lie in $\mathbb{C}^+$, the open right-half of the complex plane.

Proof: The proof is similar to that in [14], we therefore show that of (a). Consider the following Lyapunov function candidate $V(x) = x^T Px$. The derivative of $V(x)$ along the trajectories is given by

$$V(x) = x^T Pf(x,0) + \ f(x,0)^T Px \ = x^T P\left[Ax + \bar{f}(x,0)\right] + \left[x^T A^T + \bar{f}(x,0)^T\right]Px$$
$$= x^T(PA + A^T P)x + 2x^T P\bar{f}(x,0) = -x^T Qx + 2xP\bar{f}(x,0)$$

Since A is Hurwitz, $x^T Qx > 0$. For other terms, we note that $\frac{\|\bar{f}(x,0)\|_2}{\|x\|_2} \to 0$ as $\|x\|_2 \to 0$.

Therefore, for all $\gamma > 0$ there exists $r > 0$ such that $\frac{\|\bar{f}(x,0)\|_2}{\|x\|_2} < \gamma$, for all $\|x\|_2 < r$ which implies that $\|\bar{f}(x,0)\|_2 < \gamma\|x\|_2$, for all $\|x\|_2 < r$. Hence, $\dot{V}(x) < -x^T Qx + 2\gamma\|P\|_2\|x\|_2^2$, for all $\|x\|_2 < r$. Using the fact that, $0 < \lambda_{min}(Q)\|x\|_2^2 \leq x^T Qx \leq \lambda_{max}(Q)\|x\|_2^2$. It follows that

$\dot{V}(x) < -[\lambda_{min}(Q) - 2\gamma\|P\|_2]\|x\|_2^2$, for all $\|x\|_2 < r$. Choosing γ such that

$\lambda_{min}(Q) > 2\gamma\|P\|_2$ that is $\gamma < \frac{\lambda_{min}(Q)}{2\|P\|_2}$ implies $\dot{V}(x) < 0$, and therefore $x = 0$ is asymptotically stable.

4 Controllability Results

Consider system (2.3) with all its necessary assumptions

Theorem 4.1. In system (2.3), if rank $[B,\ AB] = n$, then the system (2.3) is completely Euclidean controllable on $[t_0, t_1]$.

Proof: We show that the controllability matrix W is nonsingular. Suppose not, then there exists an n-vector $v \neq 0$ such that $vWv^* = 0$. Then $\int_J [vZ(t_1, s)][vZ(t_1, s)]^* \, ds = 0$, and hence $vZ(t_1, s) = 0$ on J except possibly for a finite number of points. In particular $vZ(t_1, s) = 0$ for all s in a neighbourhood to the left of t_1. Therefore $vZ(t_1, t_1^-) = 0$ and $\frac{\partial}{\partial s}Z(t_1, t_1^-) = 0$. Using the fact that $Z(t_1, s) = X(t - s), X(0^+) = I$ and $X = 0$ for $t < 0$, it follows that $vB = 0$ and

$v[-X(0^+)B] = v[-AB] = 0$. Hence, rank $[B,\ AB] < n$, a contradiction.

Theorem 4.2. In system (2.3) assume that,

i. System (2.3) is completely Euclidean controllable and
ii. System (2.3) with $u = 0$ is uniformly asymptotically stable.

Then system (2.3) is Euclidean null controllable with constraints

Proof: By (i), system (2.3) is completely Euclidean controllable with constraints, so that $0 \epsilon\ int\ R(t_1, t_0)$, $t_1 \geq t_0$. Hence, there exists a ball S, such that $0 \epsilon\ S \leq R(t_1, t_0)$, for each $t_1 \geq t_0$. By (ii), every solution of system (2.3) with $u = 0$ satisfies $x(t, 0) \to 0$ as $t \to \infty$. Hence at some point $t_1 < \infty, x(t_1, 0) \in S$.therefore, using t_1 as an initial point and $x(t_1, 0) \equiv \omega$ as initial function, there exists a $u \in U$ and some $t_2 > t_1$ such that the solution $x(t_1, u)$ of (2.3) satisfies $x(t_2, u) = 0$, proving the theorem.

4.1 Main results

The main result of this paper will now be stated as follows;

Theorem 4.3: In system (2.3) assume that:

i. System (2.4) is uniformly asymptotically stable
ii. System (2.3) with $u = 0$ satisfies the condition $\lim_{\|x\| \to 0} \frac{\|\bar{f}(x, 0)\|}{\|x\|} = 0$
iii. rank $[B,\ AB] = n$.

Then system (2.3) is Euclidean null controllable with constraints.

Proof: By (iii), system (2.3) is completely Euclidean controllable. Hence, (i), (ii) and (iii) satisfies the requirements of Theorem 4.2 and the proof is complete.

5 Application

Here, we give the modelling of a mass spring damper system as an application. Consider a mass M attached to a nonlinear spring and damper, as illustrated in Fig. 1. The restoring force in the spring is assumed to be a function of the displacement of the mass from its equilibrium position, say $g(y)$, where y is the distance of the mass from its equilibrium position. The resistive forces due to damping is of velocity $h(\dot{y})$. Finally, u denotes the external forces acting on the mass.

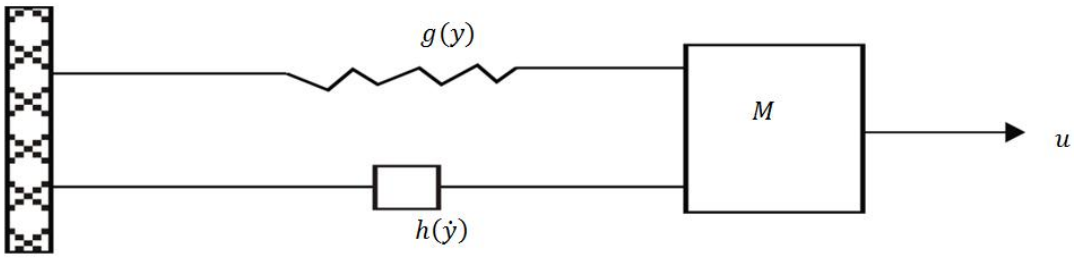

Fig. 1. A model of a mass spring damper

By Newton's law of motion the model will be given by

$$m\frac{d^2y}{dt^2} = -g(y) - h(\dot{y}) + u$$

$$m\ddot{y} = -g(y) - h(\dot{y}) + u \tag{5.1}$$

Assumptions:

i. The spring is considered to be a hard spring and modelled as a non-linear function of the form $g(y) = by + cy^2$
ii. The damper reaction force is modelled as a linear function $h(\dot{y}) = s\dot{y}$
iii. The force acting on the mass is considered the input or control variable u.

The equation of motion using Newton's law then is given by

$$m\ddot{y} + s\dot{y} + by + cy^2 = u \tag{5.2}$$

Putting (5.1) in state space form gives $\dot{x} = f(x,u)$ with $f(x,u) = Ax + \bar{f}x + Bu$, where

$$\bar{f} = \begin{bmatrix} cx_1^2 \\ 0 \end{bmatrix}^T, \; A = {}^1\!/_m \begin{bmatrix} 0 & 1 \\ -b & -s \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \\ \overline{m} \end{bmatrix}.$$

Example 1

If the nonlinear system (5.2) is estimated by the equation

$$\ddot{y} + 4\dot{y} + y^2 + 3y = u. \tag{5.3}$$

We can check the stability of the system (5.3) with $u = 0$, by applying Theorem 3.1 to get

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 - 2x_1 & -4 \end{bmatrix} \tag{5.4}$$

All points of equilibrium must lie on the real axis, when $\dot{y} = 0,\ \ddot{y} = 0$ and putting (5.3) with $u = 0$ in the state space form we get,

$$\dot{y} = x_2 = 0$$

$$\ddot{y} = -3x_1 - x_1^2 - 4x_2 = x_1^2 + 3x_1 = x_1(x_1 + 3) \ = 0$$

This implies $x_1 = 0\ or\ -3$. This gives two points of equilibrium $(x_1, x_2) = (0, 0)$ and $(x_1, x_2) = (-3,\ 0)$. The Jacobian is now evaluated at these two points. For

$$J_{(0,0)} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \text{ and } J_{(-3,\ 0)} = \begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix}$$

The linearized matrix then at equilibrium for $J_{(0,0)}$ using $det[\lambda I - J] = 0$ gives a stable point with $\lambda = -1$ or -3 and unstable point with $\lambda = -4.6$ and 0.6 for $J_{(-3,\ 0)}$. Since $J_{(0,0)}$ is stable point, the first assumption of Theorem 3.1 is satisfied i.e. $f(0,0)$ is an equilibrium point.

We now show that condition (3.4) of Theorem 3.1 is satisfied as follows. Let

$$\lim_{\|x\|\to 0} \frac{\|\bar{f}(x)\|}{\|x\|} = \lim_{\|x\|\to 0} \frac{\sqrt{(-1)^2 x_1^2 x^2}}{\sqrt{x^2}} = \lim_{x_1 \to 0} x_1 = 0$$

Thus, condition (3.4) is satisfied, hence system (5.3) is asymptotically stable. (see Fig. 2 for the phase plane diagram of system (5.3))

We now show using Theorem 4.1 that the system (5.3) is controllable, we have

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ AB = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$\text{rank}\,[B,\ AB] = rank \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix} = 2$$

Hence, system (5.3) is controllable since rank $[B,\ AB] = n = 2$ (see Fig. 3 for the open loop response of system (5.3)).

Since all the conditions in Theorem 4.3 are satisfied, we conclude that the system (5.3) is Euclidean null controllable with constraints.

Example 2

If the nonlinear system (5.2) is estimated by the system in Example 1.11of [15], given by

$$\left.\begin{aligned} \dot{y} &= u - x_1^2 \\ \ddot{y} &= -x_2 \end{aligned}\right\}. \qquad (5.5)$$

We can check the stability of the system (5.5) with $u = 0$, by applying Theorem 3.1 to get

$$J = \begin{bmatrix} -2x_1 & 0 \\ 0 & -1 \end{bmatrix}$$

At equilibrium $\dot{y} = 0,\quad \ddot{y} = 0$ and we have the point of equilibrium $(x_1, x_2) = (0, 0)$. The Jacobian is now evaluated at this point to get,

$$J_{(0,0)} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$$

The linearized matrix then at equilibrium for $J_{(0,0)}$ gives a stable point at the origin with $\lambda = 0$ or -1. Also, by condition (3.4) of Theorem 3.1; $\lim_{\|x\| \to 0} \frac{\|\bar{f}(x)\|}{\|x\|} = 0$. Thus, condition (3.4) is satisfied but the system (5.5) is marginally stable with $u = 0$.

We now check using Theorem 4.1 if the system (5.5) is controllable, we have

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ AB = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{rank}\,[B,\ AB] = rank \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1$$

Hence, system (5.5) is not controllable since rank $[B,\ AB] = n \neq 2$. Since some conditions in Theorem 4.3 are not satisfied, we conclude that the system (5.5) is not Euclidean null controllable.

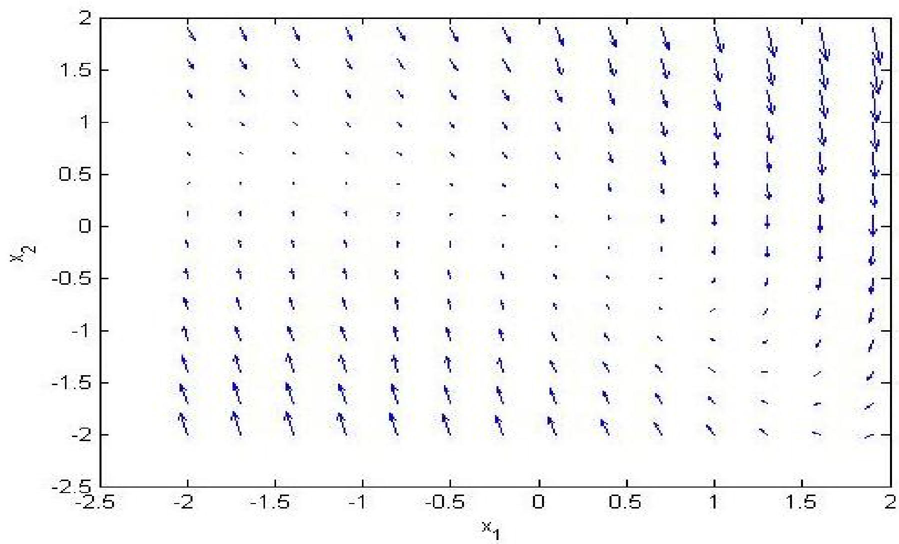

Fig. 2. Phase plane diagram of system 5.3 showing the trajectories of the Eigenvalues

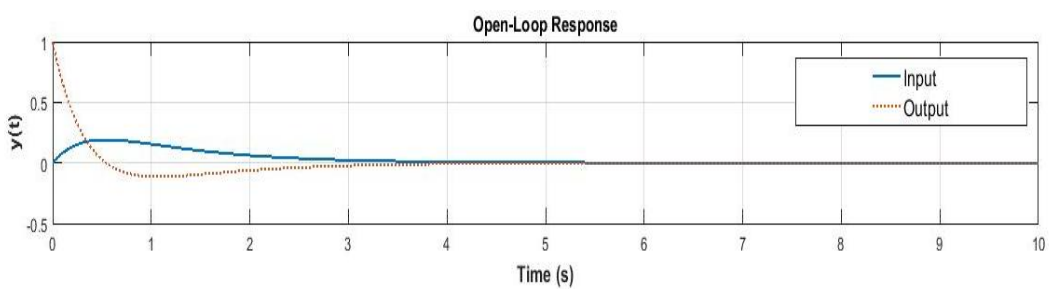

Fig. 3. Open loop output response of system 5.3

6 Discussion and Conclusion

6.1 Discussion

First the system is linearized using the Jacobian matrix as shown in equation (5.4) of the application. The asymptotic stability of the origin for the linearized system is then analyzed in terms of the eigenvalues of the system as shown in Fig. 2. The phase portrait in Fig. 2 shows some trajectories of eigenvectors of the negative eigenvalues of the linearized system (5.3) initially starting at infinite-distance, moves and converges at the critical point. That is, trajectories moves directly towards and converges to the critical points when the eigenvalues of such trajectories are less than zero. The figure also shows trajectories that are eigenvectors of the eigenvalues moving in straight line. The rest of trajectories move initially in the same direction as the eigenvector of eigenvalue with the smaller absolute value; then farther away, they bend towards the direction of the eigenvector of the eigenvalue with largest absolute value. Fig. 3 shows the open loop response of system (5.3).

6.2 Conclusion

In this paper new stability and controllability results for nonlinear systems has been obtained. The stability result was first obtained using the Lyapunov indirect method to analyze the local stability of the equilibrium point through the linearization and Jacobian matrix method. The controllability result of the systems was then obtained using the rank criterion for properness. A real-life application (mass-spring-damper system) is given to illustrate the effectiveness of the theoretical results with simulation output results using MATLAB.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Marinesson S. Stability analysis of nonlinear systems with linear programming. Gerhard-Mercatar-University, Duisburg; 2002.

[2] Davies I. Euclidean null controllability of infinite neutral differential systems. Anziam J. 2006;48(2): 285–293.

[3] Klamka J, Wyrwal J, Zawiski R. Controllability of second order dynamical systems. Bulletin of the Polish Academy of Sciences: Technical Sciences. 2017;65(3):279-295.

[4] Xie X-J, Park JH, Mukaidani H, Zhang W. Mathematical theories and applications for nonlinear control systems. Mathematical Problems in Engineering. 2019;1-6.

[5] Davies I, Haas OCL. Delay-independent closed-loop stabilization of neutral system with infinite delays. International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering. 2015;9(9):380-384.

[6] Parks PC. A. M. Lyapunov's stability theory-100 years on. IMA Journal of Mathematical Control & Information. 1992;9:275-303.

[7] Pukdeboon C. A review of fundamentals of Lyapunov theory. The Journal of Applied Science. 2011;10(2):55-61.

[8] Ibrahim Y. Global asymptotic stability of second order system of difference equations. Discrete Dynamics in Nature Society; 2008.

[9] Baranowski J, Zagorowska M, Bauer W, Dziwinski T, Piatek P. Applications of direct Lyapunov method in Caputo non-integer order systems. Elektronikair Elektrotechnika. 2015;21(2):10-13.

[10] Li YH, Tian XJ, Li XQ. Nonlinear second-order model and equilibrium point characteristic analysis of D.C traction power supply. Hindawi Publishing Corporation: ISRN Applied Mathematics. 2013;1-8.

[11] Okumus I, Soykan Y. Nature of the solutions of second order nonlinear difference equations. Journal of Progressive Research in Mathematics. 2018;14(2):23991-24071.

[12] Davies I, Haas OCL. Stability of neutral Integro-differential systems with infinite delays. Journal of the Nigerian Association of Mathematical Physics. 2019;50:13-18.

[13] Davies I, Haas OCL. Null controllability of neutral system with infinite delay. European Journal of Control Systems. 2015;26:28-34.

[14] Khalil HK. Nonlinear systems (Third Edition), Prentice Hall, Englewood Cliffs, NJ, USA; 2002.

[15] Al Hokayem P, Gallestey E. Lecture notes on nonlinear systems and control. ABB Switzerland, Ltd, Baden-Dattwil, Zurich, Switzerland; 2018.

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