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Starting Age Zero-Based Excel Automation of Optimal Policy Prescriptions and Returns for Machine Replacement Problems with Stationary Data and Age Transition Perspectives

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Aim: This investigation aimed at automating the computations of optimal replacement policies and rewards for a class of equipment replacement problems with zero starting age, based on age perspectives and stationary pertinent data.
Methodology: The aim was achieved by the exploitation of the structure of the states given as functions of the decision periods, in age-transition dynamic programming recursions.
Results: Optimal Excel interface and solution implementation templates were designed and automated for the determination of the optimal replacement policies in machine replacement problems, with pertinent data given only as a function of machine age.
Conclusion: The automation of these templates obviates the need for manual inputs of the states and stage numbering, as well as the inherent tedious and prohibitive manual computations associated with dynamic programming formulations and may be optimally exploited for sensitivity analyses on such models, paving the way for the solutions of practical and large-scale problems.

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Keywords: Age; automation; decision period; dynamic programming recursions; equipment replacement problems; feasible; optimal policy prescriptions; pertinent data; sensitivity analyses; solution template; transition diagram.

1. INTRODUCTION

The Equipment Replacement Problem is a subject of considerable and diverse research interests. Consider the problem of researching an optimal Equipment Replacement policy over an *n* - period planning horizon. At the start of each year a decision is made whether to keep the equipment in service an extra year or to replace it with a new one at some salvage value. Fan et al. [1] remarked that the primary function of equipment managers is to replace the right equipment at the time and at the lowest cost. They went on to discuss among other things, the opportunities and challenges associated with equipment replacement decision making. Fallahnezhad et al. [2], presented an optimal decision rule for minimizing total cost in designing a sampling plan for machine replacement problems using dynamic programming and Bayesian inferential approaches. The cost of replacing the machine and the cost of produced defectives were factored into the model, and the concept of control threshold policy was applied in the decision rule as follows: If the probability of producing a defective exceeded the control threshold, then the machine was replaced, otherwise the production system would be deemed to be in a state of statistical control and production would go on uninterrupted. Finally, the paper presented a numerical example as well as performed sensitivity analysis to illustrate the application of their result. Zvipore et al. [3] investigated the application of stationary equipment replacement dynamic programming model in conveyor belt replacement using a Gold mining company in Zimbabwe as a case study. Their findings revealed that conveyor belts should be replied in the mining system on a yearly basis and concluded that equipment replacement policy for conveyor belts is a necessity in a mining system, so as to achieve optimum contribution to the economic value that a mining system might accrue within a period of time. Fan et al. [4] formulated a stochastic dynamic programming-based optimization model for the equipment replacement problem that could explicitly account for the uncertainty in vehicle utilization.

As remarked by Taha [5], “the determination of the feasible values for the age of the machine at each stage is somewhat tricky”. The latter went on to obtain the optimal replacement ages using network diagrammatic approach, with machine ages on the vertical axis and decision years on the horizontal axis. In an alternative time perspective approach, Winston [6] initiated the determination process for the optimal replacement time with network diagrams consisting of upper half-circles on the horizontal axis, initiating from each feasible time of the planning horizon and terminating at feasible times, with the length of successive transition times at most, the maximum operational age of the equipment. Sequel to this, [6] formulated dynamic recursions as functions of the decision times, the corresponding feasible transition times, the problem data and the cash-flow profile. Unfortunately network diagrams are unwieldy, cumbersome and prone to errors, especially for large problem instances; consequently the integrity of the desired optimal policies may be compromised. Verma [7], Gupta & Hira [8] used the average annual cost criteria to determine alternative optimal policies and the corresponding optimal rewards in a non-dynamic programming fashion. Gress et al. [9] modeled the equipment replacement problem using a Markov decision process and a reward function that can be more helpful in processing industries. Unfortunately, the key issues of large-scale implementation and sensitivity analyses were not discussed by the afore-mentioned authors. A new impetus was provided for sensitivity analyses and implementation paradigm shift by Ukwu [10], with respect to optimal solutions to machine replacement problems. Ukwu [10] pioneered the development of computational formulas for the feasible states corresponding to each decision year in a certain class of equipment Replacement problems, thereby eliminating the drudgery and errors associated with the drawing of network diagrams for such determination. Ukwu [10] went further to design prototypical solution templates for optimal solutions to such problems, complete with an exposition on the interface and solution process. Ukwu [11] extended the formulations and results in Ukwu [10] to a class of machine replacement problems, with pertinent data given as functions of machine ages and the decision periods of the planning horizon. By restructuring the data in three – dimensional formats Ukwu [11] appropriated key features of the template in

Ukwu [10] for the extended template. Finally Ukwu [11] solved four illustrative examples of the same flavour that demonstrated the efficiency, power and utility of the solution template prototype. In Ukwu [11], it was pointed out that the template could be deployed to solve each equipment replacement problem in less than 10 percent of the time required for the manual generation of the alternate optima. However a major draw-back of the templates in Ukwu [10,11] is that for any problem instance, the inputs of the states and stage numbering are manually generated. Moreover, the templates require row updating of the formulas for the optimal criterion function values for problems of larger horizon lengths. Evidently this functionality needs to be improved upon for more speedy solution implementations, especially for practical problems of long planning horizons. Ukwu [12] used the state concept to obtain the structure of the sets of feasible replacement times corresponding to various decision times, in equipment replacement problems, thereby obviating the need for network diagrams for such determination. It went further to undertake novel formulations of the equipment replacement problems, incorporating cardinality analyses on the feasible transition states for each feasible time. Furthermore, the article designed solution implementation templates for the corresponding dynamic programming recursions. These templates circumvent the inherent tedious and prohibitive manual computations associated with dynamic programming formulations and may be optimally appropriated for sensitivity analyses on such models in just a matter of minutes. Ukwu [13] examined the effects of different planning horizons, with equipment replacement age fixed, in the Excel automated solutions in [12], to a class equipment replacement problems with stationary pertinent data. The investigation revealed that if the replacement age is fixed, and n_1 and n_2 are any two horizon lengths with $n_1 < n_2$, and $p^*_j, g(j)$ are stage j optimal decision and reward from the template with horizon length n_2, for $j \in \{n_2+1-n_1,\cdots,n_2\}$, then $p^*_j + n_1 - n_2$ *and* $g\left(j+n_1-n_2\right)$ are the corresponding optimal decision and reward in stage $j+n_1-n_2$ for the template with the horizon length n_1. Moreover the corresponding optimal rewards are equal. The results were achieved by using the structure of feasible replacement time sets and appropriate dynamic programming reward recursions. This article sets out to remedy the deficiencies in Ukwu [10,11]. The major contributions of the articles are as follows: The work provides alternative layout and solution templates to those in [10], with full automation of all computations for $t_1 = 0$. The cases $t_1 \in \{1,2,\cdots,m\}$ have been investigated by [14], where m is the mandatory equipment replacement age and t_1 is the starting age of the equipment.

The article also gives an exposition on the solution template incorporating the outputs for the given problem and general problems in that class. The outputs for the problem in Table 1 are reflected in Figs 1, 2 and 3. The outputs are consistent with the general exposition.

2. MATERIALS AND METHODS

In this section, the problem data, working definitions, elements of the DP model and the dynamic programming (DP) recursions are laid out as follows:

Equipment Starting age $= t_1$; Equipment Replacement age $= m$

S_i = The set of feasible equipment ages (states) in decision period i (say year i), $i \in \{1,2,\ldots,n\}$

$r(t)$ = annual revenue from a t- year old equipment; $c(t)$ = annual operating cost of a t- year old equipment

$s(t)$ = salvage value of a $t-$ year old equipment; $t = 0,1,\ldots,m$

I = fixed cost of acquiring a new equipment in any year

The elements of the DP are the following:

1. Stage i, represented by year $i, i \in \{1,2,\ldots,n\}$

2. The alternatives at stage (year) i. These call for keeping or replacing the equipment at the beginning of year i
3. The state at stage (year) i, represented by the age of the equipment at the beginning of year i.

Let $f_i(t)$ be the maximum net income for years $i, i+1, \ldots, n-1, n$ given that the equipment is t years old at the beginning of year i.

Note: The definition of $f_i(t)$ starting from year i to year n implies that backward recursion will be used. Forward recursion would start from year 1 to year i.

The template will implement the following theorem formulated in [6] and exploited in [10], using backward recursive procedure.

2.1 Theorem 1: Dynamic Programming Recursions for Optimal Policy and Rewards [5]

$$f_i(t) = \max \begin{cases} r(t) - c(t) + f_{i+1}(t+1); \text{ IF KEEP} \\ r(0) + s(t) - I - c(0) + f_{i+1}(1); \text{ IF REPLACE} \end{cases}$$

$$f_{n+1}(x) = s(x), \; i = 0,1,\ldots,n-1, \; x = \text{age of machine at the start of period } n+1$$

3. RESULTS AND DISCUSSION

3.1 Theorem on Analytic Determination of the Set of Feasible Ages at Each Stage [10]

Let S_i denote the set of feasible equipment ages at the start of the decision year i. Let t_1 denote the age of the machine at the start of the decision year i, that is, $S_1 = \{t_1\}$. Then for $i \in \{1, 2, \ldots, n\}$,

$$S_i = \begin{cases} \left\{\min_{2\le j\le i}\{j-1, m\}\right\} \cup \left\{1 + (i-2+t_1)\operatorname{sgn}\left(\max\{m+2-t_1-i, 0\}\right)\right\}, \text{if } t_1 < m \\ \left\{\min_{2\le j\le i}\{j-1, m\}\right\}, \text{ if } t_1 \ge m \end{cases}$$

An Excel template will now be designed and deployed to solve the practical problems below with the prescribed starting age of 0. In the sequel an exposition on the template will be given using the above problem for illustration.

Note: $t_1 = 0 \Leftrightarrow$ the process is initiated with a new machine. It is clear from theorem 3.1 that

$$t_1 = 0 \;\Rightarrow S_1 = \{0\},\, S_i = \left\{\min_{2\le j\le i}\{j-1, m\}\right\}, \text{for } \; i \in \{2, 3, \cdots, n\}$$

3.2 Application Problems on Theorem 3.1 and the Implementation of the Solution Templates

A company needs to determine the optimal replacement policy for a current t_1- year old equipment over the next n years. The following Table gives the data of the problem. The company requires that 6 - year old equipment be replaced. The cost of a new machine is $100,000.

Devise alternate optimal prescription policies with corresponding returns for the set of horizon lengths $\{8, 4\}$ and starting age $t_1 = 0$, using dynamic programming recursive approach, given the data below.

Table 1. Pertinent data for optimal policy and reward determination

Age: *t* yrs.	Revenue: *r*(*t*) ($)	Operating cost: *c*(*t*) ($)	Salvage value: *s*(*t*) ($)
0	20,000	200	-
1	19,000	600	80,000
2	18,500	1,200	60,000
3	17,200	1,500	50,000
4	15,500	1,700	30,000
5	14,00	1,800	10,000
6	12,200	2,200	5,000

This simplified expression for S_i is an integral part of the solution templates. The exposition on the solution templates implicitly explains all template outputs in the figures below.

3.3 Transition Diagram for the Alternate Optimal Policy Prescriptions

$0K1R1K2K3R1R1K2K3S$; $0K1R1K2K3R1K2K3R1S$; $0K1R1R1K2K3R1K2K3S$; $0K1K2K3R1R1R1K2K3S$; $0K1K2K3R1K2K3R1R1S$; $0K1K2K3R1K2K3R1R1S$; $0K1K2K3R1R1K2K3R1S$

Equipment Replacement Problem Solution Template					*n*	**Starting Age**	
Replacement Age =			6	**yrs**	8	0	
	Given Data			**Stage**	8		
	I **=**	100000	***V(0) = r(0)-c(0) - I =***		-80200		
Age *t* (yrs.)	0	1	2	3	4	5	6
Revenue: *r(t)* ($)	20000	19000	18500	17200	15500	14000	12200
Mnt. cost, *c(t)* ($)	200	600	1200	1500	1700	1800	2200
Salvage value, s(t)		80000	60000	50000	30000	10000	5000
K		78400	67300	45700	23800	17200	Must Replace
R		79800	59800	49800	29800	9800	4800
Opt. value: f(t)		79800	67300	49800	29800	17200	4800
Opt. Decision		*R*	*K*	*R*	*R*	*K*	*R*
State		1	2	3	4	5	6
				Stage	7		
K		85700	67100	45500	31000	17000	Must Replace
R		79600	59600	49600	29600	9600	4600
Opt. value: f(t)		85700	67100	49600	31000	17000	4600
Opt. Decision		*K*	*K*	*R*	*K*	*K*	*R*
State		1	2	3	4	5	6
				Stage	6		
K		85500	66900	46700	30800	16800	
R		85500	65500	55500	35500	15500	
Opt. value: *f(t)*		85500	66900	55500	35500	16800	
Opt. Decision		K/R	K	R	R	K	
State		1	2	3	4	5	
				Stage	5		
K		85300	72800	51200	30600		
R		85300	65300	55300	35300		
Opt. value: *f(t)*		85300	72800	55300	35300		
Opt. Decision		*K/R*	*K*	*R*	*R*		
State		1	2	3	4		

Fig. 1. Solution template for stages 8 to 5 and planning horizon length 8

				Stage	4		
K		91200	72600	51000			
R		85100	65100	55100			
Opt. value: *f(t)*		91200	72600	55100			
Opt. Decision		*K*	*K*	*R*			
State		1	2	3			
				Stage	3		
K		91000	72400				
R		91000	71000				
Opt. value: *f(t)*		91000	72400				
Opt. Decision		*K/R*	*K*				
State		1	2				
				Stage	2		
K		90800					
R		90800					
Opt. value: *f(t)*		90800					
Opt. Decision		*K/R*					
State		1					
				Stage	1		
K	110600						
R	10600						
Opt. value: *f(t)*	110600						
Opt. Decision	K						
State	0						

Fig. 2. Solution template for stages 4 to 1 of the planning horizon of length 8 problem

Optimal Return $=$ **(Maximum Net Income for years 1 to 8)** $=$ **\$110,600.00** $-$ **\$100,000.00** $=$ **\$10,600.00,** since the starting new machine is fully paid for. Note that there are seven alternate optima.

The interpretation of the selected transition diagram $0K1R1K2K3R1R1K2K3S,$ of the decision symbols K, R and $S,$ in combination with the relevant states corresponding each stage of the process leads to the following optimal policy prescription:

3.3.1 Optimal policy prescription corres pon ding to $\mathbf{0K1R1K2K3R1R1K2K3S}$

Starting with a new machine at the beginning of decision year 1, keep (deploy) the machine for one year and then replace it at the beginning of the decision year 2. Keep the replacement machine for the next two years until the beginning of the decision year 5, and then replace the machine at the beginning of each of the decision years 5 and 6. Finally keep the machine for the remaining two years until the end of the decision year 8 when it should be mandatorily salvaged.

The remaining optimal solutions can be interpreted analogously. For an n-stage process there are altogether $2(n+1)$ concatenated objects in the transition diagram initiated with 0 (the starting age) and terminated with S (the salvage symbol).

The solutions to the problem with the planning horizon length of 4 years are obtained in the solution template outputs, shown below.

3.4 Transition Diagram for the Alternate Optimal Policy Prescriptions Using the Decision Symbols $\mathbf{K, R}$ and $\mathbf{S}$:

$$0K1K2K3R1S;\ 0K1R1K2K3S$$

3.4.1 Interpretation of a selected transition diagram

$0K1K2K3R1S$: Machine Age 0 transits to 1, 2 and 3 after the machine has been deployed for 1, 2, and 3 years respectively. Then the three year-old machine is replaced and deployed for one year, whereupon the age of the machine becomes 1 at the end of year 4, noting that the age of the replacement machine at the beginning of year 4 is 0.

3.4.2 Optimal policy prescription corresponding to $0K1K2K3R1S$

Start with a new machine at the beginning of decision year 1; keep (deploy) the machine for the next three years and then replace it at the beginning of the decision year 4 until the end of the decision year 4 when it is mandatorily salvaged.

3.4.3 Optimal policy prescription corresponding to $0K1R1K2K3S$

Start with a new machine at the beginning of decision year 1; keep (deploy) the machine for one year and then replace it at the beginning of the decision year 2, with no further replacements until the end of the decision year 4 when it is mandatorily salvaged.

Equipment Replacement Problem Solution Template					*n*	**Starting Age**	
Replacement Age =			6	**yrs**	4	0	
	Given Data			**Stage**	4		
	***I* =**	100000	***V(0) = r(0)-c(0) - I =***		-80200		
Age *t* (yrs.)	0	1	2	3	4	5	6
Revenue: *r(t)* ($)	20000	19000	18500	17200	15500	14000	12200
Mnt. cost, *c(t)* ($)	200	600	1200	1500	1700	1800	2200
Salvage value, s(t)		80000	60000	50000	30000	10000	5000
K		78400	67300	45700			
R		79800	59800	49800			
Opt. value: f(t)		79800	67300	49800			
Opt. Decision		*R*	*K*	*R*			
State		1	2	3			
				Stage	3		
K		85700	67100				
R		79600	59600				
Opt. value: f(t)		85700	67100				
Opt. Decision		*K*	*K*				
State		1	2				
				Stage	2		
K		85500					
R		85500					
Opt. value: *f(t)*		85500					
Opt. Decision		K/R					
State		1					
				Stage	1		
K	105300						
R	5300						
Opt. value: *f(t)*	105300						
Opt. Decision	K						
State	0						

Fig. 3. Solution Template outputs for stages 4 to 1 of the planning horizon of length 4 problem

Optimal Return = **(Maximum Net Income for years 1 to 4)** = **$105,300.00 - $100,000.00** = **$5,300.00,** since the starting new machine is fully paid for.

3.5 An Exposition on the Solution Templates for Figs. 1, 2 and 3 and Generic Problems

Step 1: Documentation, stage numbering automation, data and fixed value storage

Use Excel references A1:H2 for documentation. Save the problem data in the indicated cells using the Copy and Paste functionality. Under the decision R, save the fixed value $V(0) = r(0) - c(0) - I$ under the fixed cell reference $F$4, using the code: = $B$6-$B$7-$C$4, <ENTER>.

Store m, n and t_1, in the fixed (absolute) cell references $D$2, $F$2, and $G$2 respectively.

To automate the stage numbering, perform the following actions:

Store last stage number n under the relative cell reference $F3, by typing: =$F$2 there, followed by <Enter>.

Secure the stage number $n-1$ under the relative cell reference $F15, by typing: =$F$2 - 1 there, followed by <Enter>.

Secure the stage number $n-2$ under the relative cell reference $F22, by typing: =$F15 - 1 there, followed by <Enter>.

Step 2: Automation of the states in all *n* stages

Blank out column B.

Type the following code in C13:

= IF (B13 >= $D$2, "", IF (B13 = MIN ($F3-1, $D$2), "", 1+B13)) <Enter>.

Click back on cell C13, position the cursor at the right edge of the cell until a crosshair appears. Then drag the crosshair across to the last the extended cell N13 to secure, to secure the stage n states and the accompanying blank spaces.

Now copy C13:N13 and paste it successively to the cell references.

$$\mathrm{C}\left[13+7\left(n-i\right)\right]:\mathrm{N}\left[13+7\left(n-i\right)\right],$$
$$\text{for } i\in\left\{n-1,\ n-2,\cdots,2,1\right\},$$

to secure the states in the remaining $n-1$ stages.

Henceforth, the act of clicking back on a specified cell, positioning the cursor at the right edge of the cell until a crosshair appears and the crosshair-dragging routine will be referred to as clerical routine/duty.

Step 3: Stage *n* Computations (Here *n* = 8)

For $t = 1$, under REPLACE, type the following code in the cell reference C10:

= IF (C13 = "","", $F$4+ $C$8+C$8) <ENTER> to secure $f_8^R(1)$.

Perform the clerical duty from cell C10 across to the extended cell N10, to secure.

$$f_8^R(2),\ \ f_8^R(3),\ \cdots f_8^R(6), \text{and blank spaces.}$$

For $t = 1$, under KEEP, type the following code in the cell reference C9:

=IF ($C13 =$D$2,"Must Replace", if (C13= "","",C$6-C$7+D$8)) <ENTER> to secure $f_8^K(1)$.

Perform the clerical from C9 across to the extended cell N9 duty to secure $f_8^K(2), f_8^K(3),\cdots, f_8^K(6)$, as well as the trailing blank spaces.

To secure $f_8(t)$, for $t\in\{1,2,\cdots,6\}$, type the following code in the cell reference C11:

=If (C13 = " "," ",if (C9 = "Must Replace", C10, max(C9,C10))) <ENTER> to secure $f_8(1)$.

Then perform the clerical routine from C11 across to N11 to secure $f_8(2), f_8(3),\cdots, f_8(6)$ and blank spaces.

3.6 Remarks on Segment Code Redundancy

In Excel, the max and min functions return values for only numeric expressions, ignoring string constants; for example if the number 5 is saved in B2 and the string constant "Must" is saved in

C2, Then in D2, the code: = max(B2, C2) <Enter> returns 5. In E2, the code: = max (B2, C2) <Enter> also returns 5. Therefore the code segment involving “if (C9 = “Must Replace”, C10” may be dispensed with throughout the template.

To obtain the optimal decision for each of the stage 8 states $t \in S_8 = \{1, 2, \cdots, 6\}$, type the following code in the cell reference C12:

=If (C13 = “”,” ”, if (C13 = $D$2, “R”, if (C9 = C10, “K/R”, if (C9 > C10, “K”, “R”)))) <ENTER> to secure $D_8(1)$.

Then perform the clerical routine to secure $D_8(2), D_8(3), \cdots, D_8(6)$ and blank spaces in sequence.

Step 4: Stage (*n* - 1) Computations (Here *n* - 1 = 7)

For $t = 1$, under REPLACE, type the following code in the cell reference C17:

=If (C20 = “”,””, $F$4+ C$8+$C11) <ENTER> to secure $f_7^R(1)$.

Perform the clerical duty to secure $f_7^R(2), f_7^R(3), \cdots, f_7^R(6)$ and succeeding blank spaces.

For $t = 1$, under KEEP, type the following code in the cell reference C16:

=If (C20 =$D$2,”Must Replace”, if (C20 = “”,””, C$6-C$7+D11)) <ENTER> to secure $f_7^K(1)$.

Perform the clerical duty to secure $f_7^K(2), f_7^K(3), \cdots, f_7^K(6)$ and succeeding blank spaces.

To secure $f_7(t)$, for $t \in \{1, 2, \cdots, 6\}$, type the following code in the cell reference C18:

=If (C20 = “ ”,” ”, if (C16 = “Must Replace”, C17, max(C16,C17)))<ENTER> to secure $f_7(1)$.

Then perform the clerical routine to secure $f_8(2), f_8(3), \cdots, f_8(6)$ and succeeding blank spaces.

To obtain the optimal decision for each of the stage 6 states $t \in S_3 = \{1, 2, \cdots, 6\}$, type the following code in the cell reference C19:

=If (C20 = “ ”,” ”,if(C20 =$D$2, “R”, if(C16 = C17, “K/R”, if (C16 > C17, “K”, “R”))))<ENTER> to secure $D_7(1)$.

Then perform the clerical routine to secure $D_7(2), D_7(3), \cdots, D_7(6)$ and the blanks.

Step 5: Stage (*n* - 2) Computations (Here *n* - 2 = 6)

Copy the contiguous region $A15:N20 of stage *n*-1 into the contiguous region $A22:N27 of stage *n*– 2 to secure stage (*n* - 2) computational values.

Step 6: Stage i Implementations, $i \in \{n-3, \cdots, 2, 1\}$, in One Fell Swoop

This is a crucial step involving a single Copy and $n-3$ Paste Operations, using the contiguous region region $A22:N27 of stage $(n-2)$.

Simply use the Copy and Paste functionality to copy and paste the contiguous region $A22:N27 successively into stages $(n-3)$ to 1 regions.

Note: Consecutive stages should be separated by a blank row. In other words, for $i \in \{n-3, n-4, \cdots, 1\}$ use the Copy and Paste functionality to copy and paste the contiguous region $A22:N27 successively into stages $(n-3)$ to 1 regions:

$$\text{A\$}\left[8+7(n-i)\right]:\text{A\$}\left[13+7(n-i)\right].$$

Step 7: Inputting the starting age $t_1 = 0$ in stage 1

Simply type in the starting age 0 in cell $B [$6+7n$] <Enter> and then copy the contiguous region $C[2+7*n*] : $C[5+7*n*] into $B[2+7*n*] :$B[5+7*n*] to complete the computational process.

Note that the stage numbering is automatically implemented, computations in all stages are automatically executed and the problem correctly solved in one fell swoop. Tremendous huh!

3.7 Remarks on the Use of the Templates for Large Problem Sizes

It is clear that the crosshair horizontal-dragging routine must be extended beyond column N, as appropriate, if $m \geq 13$. This can be optimally done before the Copy and Paste operations from stage $n-1$. Hence the template can be adequately appropriated for sensitivity analyses on this class of Equipment Replacement problems in just a matter of minutes, as contrasted with manual investigations that would at best consume hours or days with increasing values of $m \text{ and/or } n$ and the number of investigations, not to talk of the dire consequences of committing just one error in any stage computations.

4. CONCLUSION

The article designed and automated prototypical solution templates for optimal policy prescriptions for some equipment replacement problems of the stationary class, with starting age of 0, complete with an exposition on the interface and solution process. The optimal results were assured and secured by the trivial keying in of the starting age 0 in column B, in the row corresponding to stage 1 feasible state. Finally the article deployed the template to obtain alternate optimal policy prescriptions with respect to a relevant problem, with a horizon length of 8 and 4 years respectively using the starting age of 0. A long horizon length of at least 8 years may preclude attempts at manual solutions to problem instances.

Disregarding the interpretations of optimality results, it would take virtually the same amount of time to electronically implement optimal solutions to problems of any horizon length. In sharp contrast, it would take no less than forty eight hours to solve a problem of horizon length 20 or more manually. Moreover, the dire consequences of committing just one error in any stage of the process could hardly be contemplated. This would contrast quite sharply with the automated solutions that would consume no more than five minutes, subject to correct data input, demonstrating the efficiency, power and utility of the solution template prototype. In general, the template could be deployed to solve each equipment replacement problem in less than 2 percent of the time required for the manual generation of the alternate optima.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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