Open Journal of
Mathematical Sciences

PSR Press

Article

Coupled coincidence and coupled common fixed points of a pair for mappings satisfying a weakly contraction type T-coupling in the context of quasi αb-metric space

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Received: 10 March 2020; Accepted: 30 September 2020; Published: 8 October 2020.

Abstract: In this paper, we have established a theorem involving a pair of mappings satisfying a weakly contraction type condition in the context of quasi αb-metric space and proved the existence and uniqueness of coupled coincidence and coupled common fixed points. The concept of weakly compatibility of the pair of maps is applied to show the uniqueness of coupled common fixed point. This work offers an extension to the published work of Nurwahyu and Aris [1].

Keywords: Coupled coincidence point, coupled common fixed point, Quasi αb-metric space, T-coupling, weakly contraction type.

MSC: 34B15, 34B18.

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1. Introduction

Fixed point theory has been one of the most influential research topics in various fields of engineering and science. It is widely applied in solving linear algebraic equations, ordinary differential equations, integral equations, partial differential equations. The first most significant result of metric fixed point theory was given by the polish mathematician Stefan Banach, in 1922, which is known as Banach contraction principle. The famous Banach contraction principle states that in a complete metric space, a contraction self-map has a unique fixed point. It is one of the cornerstones in the development of nonlinear analysis.

The concept of b-metric spaces was introduced by Bakhtin [2] in 1989, who used it to prove a generalization of the Banach contraction principle in spaces endowed with such kind of metrics. Since then, this notion has been used by many authors to obtain various fixed point theorems. In 1993, Czewick [3] used b-metric space on his papers for their fixed point theorems on contraction mappings in the b-metric space. Then many authors also used the b-metric space for their fixed point theorems for several contraction mappings [4–8] and then other authors developed the b-metric space to become a quasi b-metric space [8,9]. The quasi b-metric space has been used on some weak contraction mappings, and the weak contraction mapping was introduced by [10]. The quasi αb-metric space was introduced by Nurwahyu [1]. It was developed from b-metric space by ignoring symmetry and modifying the triangular inequality condition of b-metric and they proposed and proved theorems which involve the existence and uniqueness of fixed point for weak contraction mappings in quasi αb-metric space.

The purpose of this study is to establish a theorem involving a pair of mappings satisfying a weakly contraction type T-coupling in the context of quasi αb-metric space and then prove the existence and uniqueness of coupled coincidence and coupled common fixed points. The concept of weakly compatibility of the pair of maps is applied to show the uniqueness of coupled common fixed point. This work is offers extension to the published work of Nurwahyu and Aris [1]. Finally, an illustrative example is presented to verify that all the conditions of the theorem are fulfilled.

2. Preliminaries

Now, we present relevant definitions and results that will be retrieved in the sequel.

Definition 1. [2] Let X be a non-empty set and $b \geq 1$ be any given real number.
Let $d : X \times X \to [0, \infty)$ be a function satisfying the following conditions:

(a) $d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$.
(b) $d(x,y) = d(y,x)$.
(c) $d(x,y) \leq b\left[d(x,z) + d(z,y)\right]$ for all $x,y,z \in X$.

Then d is known as b-metric on X and the pair (X,d) is called a b-metric space.

Definition 2. [1] Let X be a non-empty set and $0 \leq \alpha < 1$ and $b \geq 1$ be a given real number. Let $d : X \times X \to [0,\infty)$ be a function satisfying the the following conditions:

(a) $d(x,y) = d(y,x) = 0 \Leftrightarrow x = y$.
(b) $d(x,y) \leq \alpha d(y,x) + \frac{1}{2}b\left[d(x,z) + d(z,y)\right]$ for all $x,y,z \in X$.

Then d is known as quasi αb-metric on X and the pair (X,d) is called a quasi αb-metric space.

Definition 3. [12] Let (X,d) be a quasi αb-metric space and $T : X \to X$ be a self-map, then T is said to be a contraction mapping if there exists a constant $k \in [0,1)$ called a contraction factor, such that

$$d(Tx,Ty) \leq kd(x,y)$$

for all $x,y \in X$.

Definition 4. [1] Let (X,d) be a quasi αb-metric space with $0 \leq \alpha < 1$ and $b \geq 1$. A mapping $T : X \longmapsto X$ is called a weak contraction on X if there exists a function $\varphi : [0,\infty) \longmapsto [0,\infty)$, $\varphi(t) = 0$ iff $t = 0$ and satisfying the following condition:

$$d(Tx,Ty) \leq d(x,y) - \delta\varphi\left(d(x,y)\right)$$

for all $x,y \in X$ where $0 < \delta \leq 1$.

Definition 5. [1] A sequence $\{x_n\}$ in a quasi αb-metric space (X,d) is said to converge to a point $x \in X$ if and only if

$$\lim_{n\to\infty} d(x_n,x) = \lim_{n\to\infty} d(x,x_n) = 0.$$

Definition 6. [1] A sequence $\{x_n\}$ in a quasi αb-metric space (X,d) is called a Cauchy sequence if for every $\epsilon > 0$, there exists a positive integer n_0 such that for $m,n > n_0$, we have $d(x_n,x_m) < \epsilon$. That is,

$$\lim_{n,m\to\infty} d(x_n,x_m) = \lim_{n,m\to\infty} d(x_m,x_n) = 0.$$

Definition 7. [1] A quasi αb-metric space is called complete if every Cauchy sequence converges to an element in the same metric space.

Definition 8. [12] Let X be a nonempty set and $T : X \to X$ a self-map. We say that x is a fixed point of T if $Tx = x$.

Definition 9. [13] An element $(x,y) \in X \times X$, where X is any non-empty set, is called a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x,y) = x$ and $F(y,x) = y$.

Definition 10. [14]. Let (X,d) be a quasi αb-metric space and A and B be two non-empty subsets of X. Then a function $F : X \times X \to X$ is said to be a coupling with respect to A and B if $F(x,y) \in B$ and $F(y,x) \in A$ where $x \in A$ and $y \in B$.

Definition 11. [15]. Let A and B be any two non-empty subsets of a quasi αb-metric space (X,d) and $T : X \to X$ be a self-map on X. Then T is said to be SCC-Map with respect to A and B), if

(a) $T(A) \subseteq A$ and $T(B) \subseteq B$,
(b) $T(A)$ and $T(B)$ are closed in X.

Definition 12. [16] An element $(x,y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x,y) = g(x)$ and $F(y,x) = g(y)$, and (gx,gy) is called coupled point of coincidence.

Definition 13. [16] An element $(x, y) \in X \times X$, where X is any non-empty set, is called a coupled common fixed point of the mappings $F : X \times X \to X$ and and $g : X \to X$ if $F(x, y) = g(x) = x$ and $F(y, x) = g(y) = y$.

Definition 14. [16] The mappings $F : X \times X \to X$ and $g : X \to X$ are called weakly Compatible if $g(F(x, y)) = F(gx, gy)$ and $g(F(y, x)) = F(gy, gx)$ whenever $gx = F(x, y)$ and $gy = F(y, x)$.

Definition 15. A function $\omega : [0, \infty) \to [0, \infty)$ is called an altering distance function, if the following properties are satisfied:

(a) ω is monotonically non-deceasing and continuous.
(b) $\omega(t) = 0$ if and only if $t = 0$.

Theorem 1. *[1] Let (X, d) be a complete quasi αb-metric space with $0 \leq \alpha < 1$ and $b \geq 1$. Let $F : X \to X$ be a self-map satisfying the following condition:*

$$d(Fx, Fy) \leq min\Big\{d(x, Fx), d(Fy, y)\Big\} - k \cdot \omega\Big(max\Big\{d(x, Fx), d(Fy, y)\Big\}\Big)$$

for all $x, y \in X$ and $k > 0, \omega : [0, \infty) \to [0, \infty)$ is a continuous function and $\omega(t) = 0$ iff $t = 0$. Then F has a unique fixed point in X.

3. Main results

At this stage, we state our theorem and come up with the main findings.

Theorem 2. *Let A and B be any two non-empty closed subsets of a complete quasi αb-metric space (X, d) with $0 \leq \alpha < 1$, $b \geq 1$, and $k > 0$. Let $T : X \to X$ is SCC- Map on X (with respect to A and B) and $F : X \times X \to X$ be a T-coupling (with respect to A and B) if there exists an altering distance function ω such that*

$$d[F(x, y), F(u, v)] \quad \leq \quad min\Big\{d(Tx, F(x, y)), d(F(v, u), Tv)\Big\} - k \cdot \omega\Big(max\Big\{d(Tx, F(x, y)), d(F(v, u), Tv)\Big\}\Big) \quad (1)$$

for any $x, v \in A$ and $y, u \in B$, then

(i) $F(X \times X) \subseteq T(X)$
(ii) $T(A) \cap T(B) \neq \emptyset$
(iii) T and F have a coupled coincidence point in $A \times B$.
(iv) If T and F are weakly compatible, then T and F have a unique coupled common fixed point in $A \times B$.

Proof. Since A and B are non-empty subsets of X and T is a type-T coupling with respect to A and B, then for $x_0 \in A$ and $y_0 \in B$, we define the sequences $\{x_n\}$ and $\{y_n\}$ in A and B respectively such that

$$Tx_1 = F(x_0, y_0) \text{ and } Ty_1 = F(y_0, x_0).$$

This can be done because $F(X \times X) \subseteq T(X)$. Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$Tx_{n+1} = F(x_n, y_n) \text{ and } Ty_{n+1} = F(y_n, x_n) \quad (2)$$

and then, we have $d(Tx_n, Tx_{n+1}) = d\big[F(x_{n-1}, y_{n-1}), F(x_n, y_n)\big]$. Using equations (1) and (2), we have

$$\begin{aligned} d(Tx_n, Tx_{n+1}) &= d[F(x_{n-1}, y_{n-1}), F(x_n, y_n)] \\ &\leq min\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(y_n, x_n), Ty_n)\Big\} \\ &\quad - k \cdot w \Big(max\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(y_n, x_n), Ty_n)\Big\}\Big) \\ &\leq min\Big\{d(Tx_{n-1}, T(x_{n-1}, y_{n-1})), d(F(y_n, x_n), Ty_n)\Big\} \\ &\leq d(Tx_{n-1}, Fx_n). \end{aligned} \quad (3)$$

Thus, we have a non-negative and non-increasing sequence $\{Tx_n\}$. Therefore, there exists $L \geq 0$ such that $\lim_{n\to\infty} d(Tx_n, Tx_{n+1}) = L$. Since ω is continuous on $[0, \infty)$ and using (3) and for $n \to \infty$, we get $L \leq L - K \cdot \omega(L)$.

It follows that, $\omega(L) = 0$ which in turn implies that $L = 0$ since ω is an altering distance function. Similarly, we can obtain that $\lim\limits_{n\to\infty} d(Ty_n, Ty_{n+1}) = 0$.

Now, we show that $\{Tx_n\}$ and $\{Ty_n\}$ are Cauchy sequences in $T(X)$. Let $m > n \geq 1$ and using Equations (1) and (2), we have

$$\begin{aligned} d(Tx_n, Tx_m) &= d[F(x_{n-1}, y_{n-1}), F(x_{m-1}, y_{m-1})] \\ &\leq min\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(y_{m-1}, x_{m-1}), Ty_{m-1})\Big\} \\ &\quad - k\cdot w\left(max\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(y_{m-1}, x_{m-1}), Ty_{m-1})\Big\}\right) \\ &\leq min\Big\{d(Tx_{n-1}, Tx_n), d(Tx_m, Ty_{m-1})\Big\} \\ &\leq d(Tx_{n-1}, Tx_n). \end{aligned} \tag{4}$$

Similarly, we can show that

$$d(Tx_m, Tx_n) \leq d(Tx_{m-1}, Tx_m). \tag{5}$$

Taking Equations (4) and (5) as $n, m \to \infty$, we get $d(Tx_n, Tx_m) \to 0$ and $d(Tx_m, Tx_n) \to 0$. Following the same procedure as above, we can show that $d(Ty_n, Ty_m) \to 0$ and $d(Ty_m, Ty_n) \to 0$. Hence $\{Tx_n\}$ and $\{Ty_n\}$ are Cauchy sequences in $T(A)$ and $T(B)$ respectively. Since $T(A)$ and $T(B)$ are closed subset of a complete quasi αb-metric space X, we conclude that $\{Tx_n\}$ and $\{Ty_n\}$ are convergent in $T(A)$ and $T(B)$ respectively. Thus, there exist $r \in T(A)$ and $s \in T(B)$ such that

$$Tx_n \to r \text{ and } Ty_n \to s \text{ as } n \to \infty. \tag{6}$$

Using Equations (1) and (2), we have

$$\begin{aligned} d(Tx_n, Ty_n) &= d[F(x_{n-1}, y_{n-1}), F(y_{n-1}, x_{n-1})] \\ &\leq min\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(x_{n-1}, y_{n-1}), Tx_{n-1})\Big\} \\ &\quad - k\cdot w\left(max\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(x_{n-1}, y_{n-1}), Tx_{n-1})\Big\}\right) \\ &\leq min\Big\{d(Tx_{n-1}, F(x_{n-1}, y_{n-1})), d(F(x_{n-1}, y_{n-1}), Tx_{n-1})\Big\} \\ &\leq d(Tx_{n-1}, Tx_n). \end{aligned} \tag{7}$$

Similarly, we can show that

$$d(Ty_n, Tx_n) \leq d(Ty_{n-1}, Ty_n). \tag{8}$$

Taking Equations (7) and (8) as $n, m \to \infty$, we get

$$d(Tx_n, Ty_n) \to 0 \text{ and } d(Ty_n, Tx_n) \to 0. \tag{9}$$

Therefore, from (6) and (9), we have

$$s = r. \tag{10}$$

As $r \in T(A)$ and $s \in T(B) \Rightarrow s = r \in T(A) \cap T(B)$. This proves Part (i), i.e., $T(A) \cap T(B) \neq \emptyset$. Now, since $r \in T(A)$ and $s \in T(B)$, there exist $x \in A$ and $y \in B$ such that $r = T(x)$ and $s = T(y)$. From Equations (6) and (10) we have

$$Tx_n \to T(x), Ty_n \to T(y) \tag{11}$$

$$T(x) = T(y). \tag{12}$$

Using Definition 2, we have

$$d\big(r, F(x,y)\big) \leq \alpha d(F(x,y), r) + \frac{1}{2}b\big[d(r, Tx_n) + d\big(Tx_n, F(x,y)\big)\big], \tag{13}$$

and

$$d\big(F(x,y), r\big) \leq \alpha d(r, F(x,y)) + \frac{1}{2}b\big[d(Tx_n, r) + d\big(F(x,y), Tx_n\big)\big]. \tag{14}$$

Applying (1) and (2) in (13) and (14) and taking their limits as $n \to \infty$, we get

$$d\big(r, F(x,y)\big) \leq \alpha d\big(F(x,y), r\big), \tag{15}$$

and

$$d\big(F(x,y), r\big) \leq \alpha d\big(r, F(x,y)\big). \tag{16}$$

Substituting (16) into (15), we get

$$(1-\alpha^2) d\big(r, F(x,y)\big) \leq 0. \tag{17}$$

This is only possible if $d\big(r, F(x,y)\big) = 0$ since $0 \leq \alpha < 1$. Similarly, we can show that $d\big(F(x,y), r\big) = 0$. Hence $F(x,y) = r$. Moreover, we can show that $F(y,x) = s$. Hence, (Tx, Ty) is coupled point of coincedence of T and F. Now, we claim that (Tx, Ty) is the unique coupled point of coincidence of T and F. Suppose not. So, we have another coupled point of coincedence say (Tx^*, Ty^*) where $(x^*, y^*) \in X^2$ with $Tx^* = F(x^*, y^*)$ and $Ty^* = F(y^*, x^*)$. Using Equations (1) and (2), we have

$$\begin{aligned} d(Tx,Tx) &= d[F(x,y),F(x,y)] \\ &\leq min\Big\{d(Tx,F(x,y)), d(F(y,x),gy)\Big\} - k\cdot w\Big(max\Big\{d(Tx,F(x,y)), d(F(y,x),Ty)\Big\}\Big) \\ &= min\Big\{d(Tx,Tx), d(Ty,Ty)\Big\} - k\cdot w\Big(max\Big\{d(Tx,Tx), d(Ty,Ty)\Big\}\Big) \\ &\leq min\Big\{d(Tx,Tx), d(Ty,Ty)\Big\} - k\cdot w\Big(max\Big\{d(Tx,Tx), d(Ty,Ty)\Big\}\Big). \end{aligned} \tag{18}$$

From (18), it follows that $d(Tx,Tx) \leq d(Tx,Tx) - k\cdot w\Big(max\Big\{d(Tx,Tx), d(Ty,Ty)\Big\}\Big)$. Hence $d(Tx,Tx) = 0$ and $d(Ty,Ty) = 0$. Now, let us consider $d(Tx, Tx^*)$. Using Equations (1) and (2), we have

$$\begin{aligned} d(Tx,Tx^*) &= d[F(x,y),F(x^*,y^*)] \\ &\leq min\Big\{d(Tx,F(x,y)), d(F(y^*,x^*),Ty^*)\Big\} - k\cdot w\Big(max\Big\{d(Tx,F(x,y)), d(F(y^*,x^*),ty^*)\Big\}\Big) \\ &= min\Big\{d(Tx,Tx), d(Ty^*,Ty^*)\Big\} - k\cdot w\Big(max\Big\{d(Tx,Tx), d(Ty^*,Ty^*)\Big\}\Big). \end{aligned} \tag{19}$$

From (19), it follows that $d(Tx,Tx^*) \leq 0$. Hence $d(Tx,Tx^*) = 0$. Again, using Equations (1) and (2), we have

$$\begin{aligned} d(Tx^*,Tx) &= d[F(x^*,y^*),F(x,y)] \\ &\leq min\Big\{d(Tx^*,F(x^*,y^*)), d(F(y,x),Ty)\Big\} - k\cdot w\Big(max\Big\{d(Tx^*,F(x^*,y^*)), d(F(y,x),Ty)\Big\}\Big) \\ &= min\Big\{d(Tx^*,Tx^*), d(Ty,Ty)\Big\} - k\cdot w\Big(max\Big\{d(Tx^*,Tx^*), d(Ty,Ty)\Big\}\Big). \end{aligned} \tag{20}$$

From (20), it follows that $d(Tx^*,Tx) \leq 0$. Hence $d(Tx^*,Tx) = 0$. From Equations (19) and (20), we deduce that $Tx = Tx^*$. Similarly, we can show that $Ty = Ty^*$. Therefore, we have a unique coupled point of coincidence.

Now, we show that T and F have coupled common fixed point. In order to do that, we consider $d\big(Tx, F(x,x)\big)$. Using (1), we have

$$\begin{aligned} d(Tx,F(x,x)) &= d[F(x,y),F(x,x)] \\ &\leq min\Big\{d(Tx,F(x,y)), d(F(x,x),Tx)\Big\} - k\cdot w\Big(max\Big\{d(Tx,F(x,y)), d(F(x,x),Tx)\Big\}\Big) \\ &= min\Big\{d(Tx,Tx), d(F(x,x),Tx)\Big\} - k\cdot w\Big(max\Big\{d(Tx,Tx), d(F(x,x),Tx)\Big\}\Big) \\ &\leq min\Big\{0, d(F(x,x),Tx)\Big\} - k\cdot w\Big(max\Big\{0, d(F(x,x),Tx)\Big\}\Big). \end{aligned} \tag{21}$$

From (21), it follows that

$$d(Tx,F(x,x)) \leq -k\cdot w\Big(d(F(x,x),Tx)\Big). \tag{22}$$

From (22), we can deduce that $d\big(F(x,x),Tx\big)=0$ and $d\big(Tx,F(x,x)\big)=0$. Hence $Tx=F(x,x)$. Now, let $Tx=u$, then we have that $u=Tx=F(x,x)$. Since T and F are weakly compatible, we have $Tu=T(Tx)=T(F(x,x))=F(Tx,Tx)=F(u,u)$. Hence (Tu,Tu) is a coupled point of coincidence and (u,u) is a coupled coincidence point of T and F. The uniqueness of coupled point of coincidence implies that $Tu=u=Tx$. Therefore $F(u,u)=Tu=u$.That is (u,u) is the coupled common fixed point of T and F. Finally, we show the uniqueness of a coupled common fixed point of T and F. let $(u^*,u^*)\in X^2$ be another coupled common fixed point of F and T. That is, $u^*=Tu^*=F(u^*,u^*)$. Hence (Tu,Tu) and (Tu^*,Tu^*) are two coupled points of coincidence of T and F. The uniqueness of coupled point of coincidence implies that $Tu=Tu^*$ and so $F(u^*,u^*)=u^*=u$. Hence (u,u) is the unique coupled common fixed point of T and F. □

Remark 1. If we take $T=I$ (the identity map) and change the mapping $F:X\times X\to X$ to $F:X\to X$, then Theorem 2 will reduce to Theorem 1 of Nurwahyu and Aris [1].

Example 1. Let $X=[0,5]$ which is defined by $d(x,y)=|x-y|$ and $A=\{1\}$ and $B=\{1,2\}$. Then A and B are closed subsets of X. We define $F:X\times X\to X$ by $F(x,y)=min\{x,y\}$, for all $x,y\in X$. Let $T:X\to X$ be defined by

$$T(x)=\begin{cases}1 & \text{if } 0\le x<2,\\ 2 & \text{if } 2\le x\le 5.\end{cases}$$

Also, we define $\omega:[0,\infty)\to[0,\infty)$ by $\omega(t)=t^2$. Then, clearly ω is altering distances function. $T(A)=\{1\}$ and $T(B)=\{1,2\}$. So, $T(A)$ and $T(B)$ are closed subsets of a complete quasi αb-metric space $X=[0,5]$. Hence $T:X\to X$ is a SCC-Map. Now, we show that T is F-coupling with respect to A and B as $T(A)\cap B=\{1\}$ and $T(B)\cap A=\{1\}$. So, for all $x\in A$ and $y\in B$, we have $F(x,y)=1\in B$ and $F(y,x)=1\in A$, *i.e.*, $F(x,y)\in T(A)\cap B$ and $F(y,x)\in T(B)\cap A$ which show that F is a T-coupling with respect to A and B. Now, it remains to prove that F is a contractive T-coupling w.r.t. A and B. Let $x,v\in A$ and $y,u\in B$ i.e., $x=1$ and $y=1,2$. Four cases will arise for y and u.

Case (i): $x=v=1$ and $y=u=1$.
Case (ii): $x=v=1$ and $y=1,u=2$.
Case (iii): $x=v=1$ and $y=2,u=1$.
Case (iv): $x=v=1$ and $y=u=2$.

For **Case (i)**, i.e., $x=v=1$ and $y=u=1$, we have $F(x,y)=F(u,v)=F(v,u)=F(1,1)=1$, $T(x)=T(v)=T(1)=1$, $d(1,1)=0$, and

$$\begin{aligned} d[F(x,y),F(u,v)] &\le min\Big\{d(Tx,F(x,y)),d(F(v,u),Tv)\Big\}-k\cdot\omega\Big(max\Big\{d(Tx,F(x,y)),d(F(v,u),Tv)\Big\}\Big),\\ d(0) &\le 0-k\cdot\omega(0),\\ 0 &\le 0,\end{aligned}$$

which proves Case (i).

In a similar fashion, we can show for the other three cases. Hence, T and F satisfy all the conditions of Theorem 2. Thus T and F have a strong coupled fixed points in $A\cap B$. Clearly $T(A)\cap T(B)=\{1\}\neq\emptyset$. 1 is the unique strong coupled coincidence point and (1, 1) is the unique coupled common fixed point of T and g in $A\cap B$ as $T(1)=F(1,1)=min\{1,1\}=1$.

4. Conclusion

In this paper, we have established a theorem involving a pair of mappings satisfying a weakly contraction type T-coupling in the context of quasi αb-metric space and then prove the existence and uniqueness of coupled coincidence and coupled common fixed points. The concept of weakly compatibility of the pair of maps is applied to show the uniqueness of coupled common fixed point. We also provide an example in support of our main result. Our work extended the published work of Nurwahyu and Aris [1].

Acknowledgments: The authors want to thank the anonymous referee for the throughout reading of the manuscript and several suggestions that help us improve the presentation of the paper. We would also like to thank the College of Natural Sciences, Jimma University for funding this staff research project.

Author Contributions: All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Conflicts of Interest: "The authors declare no conflict of interest."

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