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Mathematical Model of Formation Aerosol Particles Fractal Storm Cloud

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Short Research Article

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Abstract

The paper proposes a new educational model of aerosol particles in thunderclouds. The model takes into account the fractal properties of the storm clouds, and its solution was obtained by numerical methods of fractional calculus. Built new profiles calculated curves that are consistent with the classical Lifshitz-Slezov-Wagner theory.

Keywords: Coalescence; lifshitz-slezov-wagner theory; fractal dimension; the mathematical model.

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1 Introduction

Formation aerosol particles is an important process in the formation of thunderstorm electricity and studied by the theory of Lifshitz-Slyozov-Wagner (LSW) [1]-[2]. This process is referred to as coalescence. According to the theory LSV distribution of aerosols in size depending on the time of power law: $R(t) \sim t^{\frac{1}{3}}$ in the case of the diffusion mechanism of formation [1], according to the law $R(t) \sim t^{\frac{1}{2}}$ in the case of the reaction on the surface of aerosol [1].

It is known that power laws describe well the fractal objects and processes [3]. In [4], the authors, using the experimental data, was found fractal dimension hail clouds, which has a value of $d_f = 1,36 \pm 0,1$. Therefore it can be concluded that the storm clouds have fractal properties that can be described by the mathematical apparatus of fractional calculus [5].

2 Statement of the Problem

Consider the scheme coalescence LSW when fall grains (aerosols) are fixed and grown by diffusion from the surrounding solution [6]. In determining the concentration of the bulk amount of the substance dissolved in a unit volume of solution, the diffusion flux is determined by the formula

$$q(r,t) = \lambda \frac{\partial u(r,t)}{\partial r}, \tag{2.1}$$

when $\lambda > 0$ – the diffusion coefficient of solute, $u(r,t)$ – concentration of the saturated solution, r – the aerosol radius, $t > 0$ – time.

Diffusion flux (2.1) at the surface of the aerosol ($r = R$) coincides with the rate of change of its radius:

$$\frac{dR(t)}{dt} = \lambda \frac{\partial u(r,t)}{\partial r}|_{r=R}. \tag{2.2}$$

In [6], taking into account the critical radius $R_c(t)$ and the relation (2.2), we obtain the equation:

$$\frac{dR(t)}{dt} = \frac{R_c^3(0)}{R(t)} \left(\frac{1}{R_c(t)} - \frac{1}{R_c(t)} \right), \tag{2.3}$$

when $R_c^3(0)$ – the value of critical radius, it is responsible for the beginning stages of coalescence. In the formula (2.3) with values $R(t) > R_c(t)$ of the aerosol is increased, and with - $R(t) < R_c(t)$ – dissolved.

In the case of the fractal structure of a thundercloud diffusion flux (2.1) can be generalized by the formula [5]

$$q(r,t) = \lambda \partial_{0t}^{\alpha} u(r,\tau), 0 < \alpha < 1, \tag{2.4}$$

when $\partial_{0t}^{\alpha} u(r,\tau) = \frac{1}{\Gamma(1-\alpha)} \int\limits_0^t \frac{u'_\tau(r,\tau)\, d\tau}{(t-\tau)^{\alpha}}$ – fractional differentiation operator Gerasimov-Caputo order [7]; $\Gamma(x)$ – Euler gamma-function. Taking into account (2.4), in [8] proposed a generalization of equation (2.3), which can be written as:

$$\partial_{0t}^{\alpha} R(\tau) = \frac{R_c^3(0)}{R(t) R_c(t)} \left(1 - \frac{R_c(t)}{R(t)} \right), \tag{2.5}$$

We solve equation (2.5) using an explicit finite-difference scheme. We introduce a uniform grid with a constant pitch τ in the time interval from 0 to T , where T – any positive integer. Then we have the following approximation [9]

$$\partial_{0t}^{\alpha} R(\tau) \approx B \sum_{k=0}^{j-1} b_k (R_{j-k+1} - R_{j-k}), j = 1, .., N \tag{2.6}$$

when $B = \dfrac{\tau^{-\alpha}}{\Gamma(2-\alpha)}$, N - number of points and $b_k = (k+1)^{1-\alpha} - k^{1-\alpha}$. Using (2.6) of the formula (2.5), we obtain

$$R_{j+1} = R_j - \sum_{k=0}^{j-1} b_k \left(R_{j-k+1} - R_{j-k}\right) + \frac{R_c^3}{BR_{cj}R_j}\left(1 - \frac{R_{cj}}{R_j}\right). \tag{2.7}$$

Equation (2.7) is a numerical solution of the equation formation - dissolution of aerosol in the generalized theory of coalescence LSW fractal thundercloud.

3 Results and Discussion

Numerical modeling was performed using Maple [10], and the values of the parameters in the model were chosen according to [11].

Fig. 1 shows the calculated curves found by the formula (2.7), which describe the variation the aerosol radius according to the parameter values In the framework of the generalized theory of Lifshitz-Slyozov-Wagner.

Note that when $\alpha = 1$ generalized theory LSW is the classical theory of LSW (red line in Fig. 1). In the general case of Fig. 1 shows that with decreasing the parameter is the rapid growth of the aerosol (the theory of Lifshitz-Slezov), and its dissolution (Wagner theory) on the contrary slows down. This effect is related to the fractal properties of the medium, namely with a power law distribution of aerosols in size depending on the time [4] and the effects of "memory" in the medium.

The calculated curves of Fig. 1, to some extent, are similar to the calculated curves of the radius of crystals in the framework of LSW (Fig. 2a), and especially on the calculated curves in the theory of "communicating neighbors" (Fig. 2b), presented in [11].

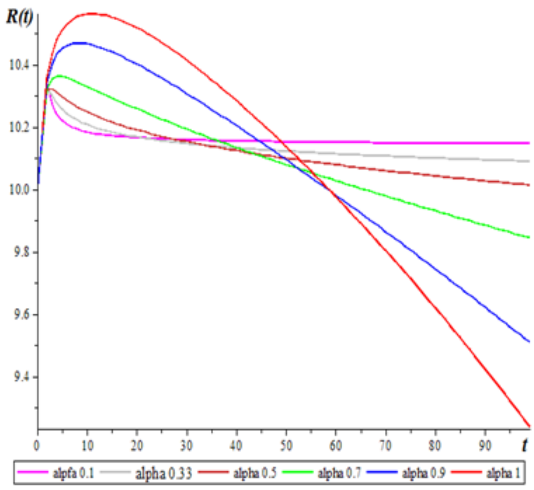

Fig. 1. Calculated curves the aerosol radius, constructed according to the decision (2.7) of the generalized model LSW depending on the parameter values α

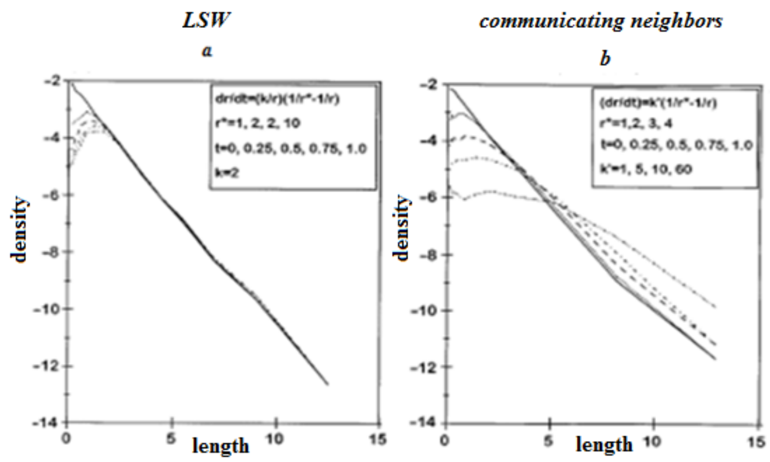

Fig. 2. The calculated curves of the radius of crystals, presented in [11] *a* – classical theory LSW; *b* - the theory of "communicating neighbors"

4 Conclusions

Indeed, in the theory of "communicating neighbors" in the generalized theory of LSV accounted for nonlocal effects in time or "memory" effect, so the calculated curves behave similarly. On this basis, we can conclude that it makes sense to continue the further development and study of the properties of the generalized theory of LSW.

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Competing Interests

The authors declare that no competing interests exist.

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