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COMPUTING DEGREE-BASED TOPOLOGICAL INDICES OF JAHANGIR GRAPH

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Abstract. Topological indices are numerical numbers associated with a graph that helps to predict many properties of underlined graph. In this paper we aim to compute multiplicative degree based topological indices of Jahangir graph.

Index Terms: Zagreb index; Randić index; polynomial; degree; graph.

1. Introduction

The study of topological indices, based on distance in a graph, was effectively employed in 1947, in chemistry by Weiner [1]. He introduced a distance-based topological index called the *Wiener index* to correlate properties of alkenes and the structures of their molecular graphs. Topological indices play a vital role in computational and theoretical aspects of chemistry in predicting material properties [2, 3, 4, 5, 6, 7, 8]. Several algebraic polynomials have useful applications in chemistry [9, 10].
A graph G is an ordered pair (V, E), where V is the set of vertices and E is the set of edges. A path from a vertex v to a vertex w is a sequence of vertices and edges that starts from v and stops at w. The number of edges in a path is called the length of that path. A graph is said to be connected if there is a path between any two of its vertices.
The distance $d(u, v)$ between two vertices u, v of a connected graph G is the length of a shortest path between them. Graph theory is contributing a lion's share in many areas such as chemistry, physics, pharmacy, as well as in industry

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[11]. We will start with some preliminary facts.
The first and second multiplicative Zagreb indices [12] are defined as

$$MZ_1(G) = \prod_{u \in V(G)} (d_u)^2,$$

$$MZ_2(G) = \prod_{uv \in E(G)} d_u.d_u,$$

and the Narumi-Kataymana index [13] is defined as

$$NK(G) = \prod_{u \in V(G)} d_u,$$

Like the Wiener index, these types of indices are the focus of considerable research in computational chemistry [14, 15, 16, 17]. For example, in 2011, Gutman [14] characterized the multiplicative Zagreb indices for trees and determined the unique trees that obtained maximum and minimum values for $M_1(G)$ and $M_2(G)$. Wang *et al.* [17] defined the following index for k-trees,

$$W_1^s(G) = \prod_{u \in V(G)} (d_u)^s.$$

Notice that $s = 1, 2$ is the Narumi-Katayama and Zagreb index, respectively. Based on the successful consideration of multiplicative Zagreb indices, Eliasi *et al.* [18] continued to define a new multiplicative version of the first Zagreb index as

$$MZ_1^*(G) = \prod_{uv \in E(G)} (d_u + d_u),$$

Furthering the concept of indexing with the edge set, the first author introduced the first and second hyper-Zagreb indices of a graph [19]. They are defined as

$$HII_1(G) = \prod_{uv \in E(G)} (d_u + d_u)^2,$$

$$HII_2(G) = \prod_{uv \in E(G)} (d_u.d_u)^2,$$

In [20] Kulli *et al.* defined the first and second generalized Zagreb indices

$$MZ_1^a(G) = \prod_{uv \in E(G)} (d_u + d_u)^a,$$

$$MZ_2^a(G) = \prod_{uv \in E(G)} (d_u.d_u)^a,$$

Multiplicative sum connectivity and multiplicative product connectivity indices [21] are define as:

$$SCII(G) = \prod_{uv \in E(G)} \frac{1}{(d_u + d_u)},$$

$$PCII(G)(G) = \prod_{uv \in E(G)} \frac{1}{(d_u.d_u)}.$$

Multiplicative atomic bond connectivity index and multiplicative Geometric arithmetic index are defined as

$$ABCII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_u + d_u - 2}{d_u.d_u}},$$

$$GAII(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_u.d_u}}{d_u + d_u},$$

$$GA^aII(G) = \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_u.d_u}}{d_u + d_u}\right)^a. \tag{1}$$

In this paper we compute multiplicative indices of Jahangir graphs. The Jahangir graph $J_{m,n}$ is a graph on $nm+1$ vertices and $m(n+1)$ edges for all $n \geq 2$ and $m \geq 3$. $J_{m,n}$ consists of a cycle C_{mn} with one additional vertex which is adjacent to m vertices of C_{nm} at distance to each other. Figure 1 shows some particular cases of $J_{m,n}$.

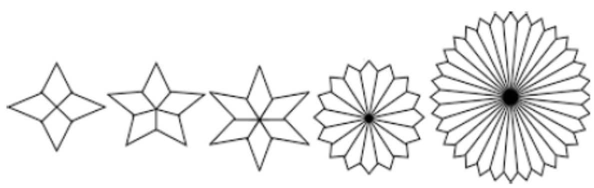

Figure 1. Jahangir graph.

2. Computational Results

In this section, we present our computational results.

Theorem 2.1. *Let $J_{m,n}$ be the jahangir's graph. Then*

(1) $MZ_1^a(J_{m,n}) = (4)^{am(n-2)} \times (5)^{2am} \times (3+m)^{am}$,
(2) $MZ_2^a(J_{m,n}) = (4)^{am(n-2)} \times (6)^{2am} \times (3m)^{am}$,
(3) $G^aAII(J_{m,n}) = \left(\frac{2\sqrt{6}}{5}\right)^{2am} \times \left(\frac{2\sqrt{3\times m}}{3+m}\right)^{am}$.

Proof. Let G be the graph of $J_{m,n}$. It is clear that the total number of vertices in $J_{m,n}$ are $8n+2$ and total number of edges are $10n+1$ The edge set of $J_{m,n}$ has following three partitions,

$$E_1 = E_{2,2} = \{e = uv \in E(J_{m,n}) : d_u = 2, d_v = 2\},$$
$$E_1 = E_{2,3} = \{e = uv \in E(J_{m,n}) : d_u = 2, d_v = 3\}.$$

and

$$E_1 = E_{3,m} = \{e = uv \in E(J_{m,n}) : d_u = 3, d_v = m\}.$$

Now,

$$| E_1(J_{m,n}) |= m(n-2).$$

$$| E_2(J_{m,n}) |= 2m,$$

and

$$| E_1(J_{m,n}) |= m.$$

(1)

$$\begin{aligned}
& MZ_1^a(J_{m,n}) \\
= & \prod_{uv\in E(G)} (d_u + d_v)^a \\
= & \prod_{uv\in E_1(J_{m,n})} (d_u + d_v)^a + \prod_{uv\in E_2(J_{m,n})} (d_u + d_v)^a + \prod_{uv\in E_3(J_{m,n})} (d_u + d_v)^a \\
= & (d_u + d_v)^{a|E_1(J_{m,n})|} + (d_u + d_v)^{a|E_2(J_{m,n})|} + (d_u + d_v)^{a|E_3(J_{m,n})|} \\
= & (2+2)^{am(n-2)} + (2+3)^{a(2m)} + (3+m)^{am} \\
= & (4)^{am(n-2)} \times (5)^{2am} \times (3+m)^{am}.
\end{aligned}$$

(2)

$$\begin{aligned}
& MZ_2^a(J_{m,n}) \\
= & \prod_{uv\in E(G)} (d_u.d_v)^a \\
= & \prod_{uv\in E_1(J_{m,n})} (d_u.d_v)^a + \prod_{uv\in E_2(J_{m,n})} (d_u.d_v)^a + \prod_{uv\in E_3(J_{m,n})} (d_u.d_v)^a \\
= & (d_u.d_v)^{a|E_1(J_{m,n})|} + (d_u.d_v)^{a|E_2(J_{m,n})|} + (d_u.d_v)^{a|E_3(J_{m,n})|} \\
= & (2.2)^{am(n-2)} + (2.3)^{a(2m)} + (3.m)^{am} \\
= & (4)^{am(n-2)} \times (6)^{2am} \times (3m)^{am}.
\end{aligned}$$

(3)

$$\begin{aligned}
& G^a AII(J_{m,n}) \\
= & \prod_{uv\in E(G)} \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^a \\
= & \prod_{uv\in E(G)} \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^a + \prod_{uv\in E(G)} \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^a + \prod_{uv\in E(G)} \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^a \\
= & \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^{a|E_1(J_{m,n})|} \times \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^{a|E_2(J_{m,n})|} \times \left(\frac{2\sqrt{d_u d_v}}{d_u + d_v}\right)^{a|E_3(J_{m,n})|}
\end{aligned}$$

$$
\begin{aligned}
&= \left(\frac{2\sqrt{2.2}}{2+2}\right)^{am(n-2)} \times \left(\frac{2\sqrt{2.3}}{2+3}\right)^{a(2m)} \times \left(\frac{2\sqrt{3.m}}{3+m}\right)^{am} \\
&= \left(\frac{2\sqrt{6}}{5}\right)^{2am} \times \left(\frac{2\sqrt{3\times m}}{3+m}\right)^{am}.
\end{aligned}
$$

□

Corollary 2.2. *Let $J_{m,n}$ be the Jahangir's graph. Then*

(1) $MZ_1(J_{m,n}) = (4)^{m(n-2)} \times (5)^{2m} \times (3+m)^m$,
(2) $MZ_2(J_{m,n}) = (4)^{m(n-2)} \times (6)^{2m} \times (3m)^m$,
(3) $GAII(J_{m,n}) = \left(\frac{2\sqrt{6}}{5}\right)^{2m} \times \left(\frac{2\sqrt{3\times m}}{3+m}\right)^{m}$.

Proof. We get our result by putting $\alpha = 1$ in the Theorem 2.1. □

Corollary 2.3. *Let $J_{m,n}$ be the Jahangir's graph. Then*

(1) $HII_1(J_{m,n}) = (4)^{2m(n-2)} \times (5)^{4m} \times (3+m)^{2m}$,
(2) $HII_2(J_{m,n}) = (4)^{2m(n-2)} \times (6)^{4m} \times (3m)^{am}$.

Proof. We get our desired results by putting $\alpha = 2$ in Theorem 2.1. □

Corollary 2.4. *Let $J_{m,n}$ be the Jahangir's graph. Then*

(1) $XII(J_{m,n}) = \left(\frac{1}{2}\right)^{m(n-2)} \times \left(\frac{1}{\sqrt{5}}\right)^{2m} \times \left(\frac{1}{\sqrt{m+1}}\right)^{mn}$,
(2) $\chi II(J_{m,n}) = \left(\frac{1}{2}\right)^{m(n-2)} \times \left(\frac{1}{\sqrt{6}}\right)^{2m} \times \left(\frac{1}{\sqrt{m+1}}\right)^{mn}$.

Proof. We get our desired results by putting $\alpha = \frac{-1}{2}$ in Theorem 2.1. □

Theorem 2.5. *Let $J_{m,n}$ be the Jahangir's graph. Then*

$$ABCII(J_{m,n}) = \left(\frac{1}{\sqrt{2}}\right)^{mn} \times \left(\sqrt{\frac{m+1}{3m}}\right)^{mn}.$$

Proof. By using the edge partition of the Jahangir graph given in Theorem 2.1

$$
\begin{aligned}
ABCII(J_{m,n}) &= \prod_{uv\in E(J_{m,n})} \sqrt{\frac{d_u+d_u-2}{d_u.d_u}} \\
&= \prod_{uv\in E_1(J_{m,n})} \sqrt{\frac{d_u+d_u-2}{d_u.d_u}} \times \prod_{uv\in E_2(J_{m,n})} \sqrt{\frac{d_u+d_u-2}{d_u.d_u}} \\
&\quad \times \prod_{uv\in E_3(J_{m,n})} \sqrt{\frac{d_u+d_u-2}{d_u.d_u}} \\
&= \left(\sqrt{\frac{d_u+d_u-2}{d_u.d_u}}\right)^{|E_1(J_{m,n})|} \times \left(\sqrt{\frac{d_u+d_u-2}{d_u.d_u}}\right)^{|E_2(J_{m,n})|}
\end{aligned}
$$

$$
\begin{aligned}
& \times\left(\sqrt{\frac{d_u+d_u-2}{d_u.d_u}}\right)^{|E_3(J_{m,n})|} \\
= \quad & \left(\sqrt{\frac{d_u+d_u-2}{d_u.d_u}}\right)^{m(n-2)} \times \left(\sqrt{\frac{d_u+d_u-2}{d_u.d_u}}\right)^{2m} \\
& \times\left(\sqrt{\frac{d_u+d_u-2}{d_u.d_u}}\right)^{m} \\
= \quad & \left(\sqrt{\frac{1}{2}}\right)^{m(n-2)} \times \left(\sqrt{\frac{1}{2}}\right)^{2m} \times \left(\sqrt{\frac{m+1}{3m}}\right)^{mn} \\
= \quad & \left(\frac{1}{\sqrt{2}}\right)^{mn} \times \left(\sqrt{\frac{m+1}{3m}}\right)^{mn}
\end{aligned}
$$

$\square$

Competing Interests

The authors declare that they have no competing interests.

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