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The Existence of Random Attractor for Stochastic Suspension Bridge Equation

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

This paper is aimed at an extensible suspension bridge equation with additive noise and linear memory. For the suspension bridge equations without additive noise and memory, there are many classical results. However, the extensible suspension bridge equations with both additive noise and linear memory were not studied before. The object of this paper is to provide a result on the random attractor to the above problem using compactness translation theorem and decomposition technique.

Keywords: Suspension bridge equations; random attractors; linear memory; additive noise.

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1 Introduction

Denote $(\Omega, F, \mathbb{P})$ be a probability space, where

$$\Omega = \{\omega = (\omega_1, \omega_2, \cdots, \omega_m) \in C(\mathbb{R}, \mathbb{R}^m) : \omega(0) = 0\},$$

is endowed with compact open topology, F is the $\mathbb{P}$-completion of Borel σ-algebra on Ω, and $\mathbb{P}$ is the corresponding Wiener measure. Define the time shift as

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), \qquad t \in \mathbb{R},\ \omega \in \Omega.$$

Therefore, $(\Omega, F, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ is an ergodic metric dynamical system.

In this paper, we study the existence of random attractors for the following suspension bridge equations with memory and additive noise:

$$\begin{cases} u_{tt} + \Delta^2 u + \Delta^2 u_t + ku^+ + (p - \beta\|\nabla u\|^2_{L^2(U)})\Delta u + \int_0^\infty \mu(s)\Delta^2(u(t) \\ \qquad\qquad - u(t-s))ds = g(x) + \sum_{j=1}^m h_j \dot{W}_j,\ x \in U, t > \tau, \\ u(x,t) = \Delta u(x,t) = 0, \qquad x \in \partial U,\ t \le \tau,\ \tau \in \mathbb{R}, \\ u(x,\tau) = u_0(x), u_t(x,\tau) = u_1(x), \qquad x \in U, \end{cases} \tag{1.1}$$

where U is a bounded open set in $\mathbb{R}^2$, as well as a smooth boundary ∂U, $u = u(x,t)$ is a real function on $U \times [\tau, +\infty)$ and accounts for the downward deflection of the bridge in the vertical plane, u^+ stands for its positive part, namely,

$$u^+ = \begin{cases} u, & \text{if } u \ge 0, \\ 0, & \text{if } u \le 0. \end{cases}$$

$k > 0$ denotes the spring constant, p is negative when the bridge is stretched, positive when compressed, $h_j(x) \in H_0^2(U) \cap H^4(U)$, $(j = 1, 2, 3, \cdots, m)$, $\{W_j\}_{j=1}^m$ are Wiener processes on $(\Omega, F, \mathbb{P})$. We denote

$$\omega(t) = (W_1(t), W_2(t), \cdots, W_m(t)), t \in \mathbb{R}.$$

The function $\mu(s)$ and $g(x)$ satisfy:

$(H_1):\ \mu(s) \in C^1(\mathbb{R}^+) \cap L^2(\mathbb{R}^+),\ \mu(s) \ge 0,\ \mu'(s) + \delta\mu(s) \le 0,\ \forall s \in \mathbb{R}^+$ and $\delta > 0$.
$(H_2):\ g \in H_0^1 \cap H^2(U)$.

Following Dafermos [1], we introduce a Hilbert "history" space

$$\mathfrak{R}_{\mu,2} = L^2_\mu(\mathbb{R}^+, H^2(U) \cap H_0^1(U)),$$

where

$$(\eta_1, \eta_2)_{\mu,2} = \int_0^\infty \mu(s)\,(\Delta\eta_1(s), \Delta\eta_2(s))\,ds, \quad \forall \eta_1, \eta_2 \in \mathfrak{R}_{\mu,2}.$$

We introduce new variables

$$\eta(t,x,s) = u(t,x) - u(t-s,x).$$

To facilitate easy calculation, we take $\beta = 1$. Then set $E = (H^2(U) \cap H_0^1(U)) \times L^2(U) \times \mathfrak{R}_{\mu,2}$, $Z = (u, u_t, \eta)^T$, then the system (1.1) is equivalent to initial value problem below in the Hilbert space E :

$$\begin{cases} Z_t = L(Z) + N(Z, t, W(t)), & x \in U,\ t \ge \tau,\ s \in \mathbb{R}^+, \\ Z(\tau) = Z_\tau = (u_0(x), u_1(x), \eta_0(x,s)), & (x,s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.2}$$

where

$$\begin{cases} u(t,\tau,x) = \eta(t,\tau,x,s) = \eta(t,\tau,x,0) = 0, \ x \in \partial U, \ t > \tau, \ s \in \mathbb{R}^+, \\ \Delta u(t,\tau,x) = \Delta\eta(t,\tau,x,s) = \Delta\eta(t,\tau,x,0) = 0, \ x \in \partial U, \ t \le \tau, \ s \in \mathbb{R}^+, \\ u(\tau,x) = u(\tau,\tau,x) = u_0(\tau,x), \ u_t(\tau,x) = u_t(\tau,\tau,x) = u_1(x), \qquad x \in U, \\ \eta(\tau,x,s) = \eta_0(x,s) = u(\tau,x) - u(\tau-s,x), \qquad (x,s) \in U \times \mathbb{R}^+, \end{cases} \tag{1.3}$$

$$L(Z) = \begin{pmatrix} u_t \\ -\Delta^2 u - \Delta^2 u_t - \int_0^\infty \mu(s)\Delta^2\eta(s)ds \\ u_t - \eta_s \end{pmatrix}, \tag{1.4}$$

$$N(Z,t,W(t)) = \begin{pmatrix} 0 \\ -ku^+ - (p - \|\nabla u\|^2_{L^2(U)})\Delta u + \sum_{j=1}^m h_j \dot{W}_j \\ 0 \end{pmatrix}, \tag{1.5}$$

$$D(L) = \left\{ Z \in E \mid \begin{array}{c} u + \int_0^\infty \mu(s)\Delta^2\eta(s)ds \in H^3(U) \cap H_0^2(U), \\ u_t \in H_0^2(U), \eta(s) \in H_\mu^1(\mathbb{R}^+, H^2(U) \cap H_0^1(U)), \eta(\tau) = 0 \end{array} \right\}, \tag{1.6}$$

here $H_\mu^1(\mathbb{R}^+, H^2(U) \cap H_0^1(U)) = \{\eta : \eta(s), \partial_s\eta(s) \in L_\mu^2(\mathbb{R}^+, H^2(U) \cap H_0^1(U))\}$.

The suspension bridge equations are an important mathematical model in engineering. Lazer and McKenna [2] investigated the problem of nonlinear oscillation in a suspension bridge. Lately, similar models have been considered by many authors, most of them concentrating on the existence and decay estimates of solutions; see [3, 4, 5] and references therein.

As $h_j(1 \le j \le m) = 0$ and $p = \beta = \mu = 0$, Eq. (1.1) reduces to a normal determined suspension bridge equation. There were many publications concerning the existence of their uniform attractors, pullback attractors and exponential attractors, see for instance [6, 7, 8, 9, 10, 11] and reference therein. The asymptotic behavior of suspension bridge equation with past history (i.e. $\mu \neq 0$) has been studied by many authors when $p = \beta = 0$ (see [12, 13, 14]). When $h_j(1 \le j \le m) = 0$, p, β and μ are not zero, the equation is so called a deterministic Kirchhoff type problem. In this case, existence of the attractors in bounded domains has been studied in [15].

In the case when $h_j(1 \le j \le m) \neq 0$, (1.1) is just the stochastic suspension bridge equation that we are concerned with in this paper. In [16], the authors proved the existence of random attractors on a bounded domain as $p = \beta = 0$. Ma and Xu [17] studied the random attractors of suspension bridge equation with noise as $\mu = 0$. As we know, there is no results on the asymptotic behavior of the stochastic suspension bridge equations (1.1). Therefore, we will focus on random attractors for (1.1).

To gain the existence of random attractors for a random dynamical system, without loss of generality, the important step is to obtain the compactness of the system in some sense. For our system (1.1), there are three essential difficulties in showing the compactness. The first difficulty is that the geometric nonlinearity, it makes our estimates more complex, so we need to make more accurate calculation. The second difficulty is that the parameter $p \in \mathbb{R}$ also brings some difficulties. The third difficulty is that the memory kernel itself: for one thing, compact embedding does not exist in the history space; for another thing, the finite rank method can not used. In order to move on these obstacles, we have to introduce a new variable and define a improved Hilbert space, along with the compactness translation theorem based on the methods of [18]. Finally, by constructing some functionals and using the decomposition technique, we arrive at our aims, that is, the existence of random attractors to (1.1) is showed.

Arrangement throughout the paper is as follows. Background materials on random dynamical system and random attractors are iterated in Section 2. In Section 3, we show that (1.1) generates a continuous random dynamical system. In Section 4, we establish the existence of the uniform absorbing set. In Section 5, we decompose the solution into two parts: one part decays exponentially and another part is bounded. Finally, we show the existence of random attractors.

2 Random Dynamical Systems

In this section, we give some concepts on random dynamical system and a theorem for random dynamical system in [19], which are crucial for obtaining our main results.

Suppose $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ-algebra $B(X)$ and $\big(\Omega, F, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}}\big)$ be a metric dynamical system.

Definition 2.1. Let $(\Omega, F, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$ be a metric dynamical system. Suppose that the mapping $\phi : \mathbb{R}^+ \times \Omega \times X \to X$ is $(B(\mathbb{R}^+) \times F \times B(X), B(X))$-measurable and satisfies the following properties:

(i) $\phi(0, \omega)x = x$;
(ii) $\phi(s, \theta_t\omega) \circ \phi(t, \omega)x = \phi(s + t, \omega)x$;

for all $s, t \in \mathbb{R}^+,\ x \in X$ and $\omega \in \Omega$. Then ϕ is called a random dynamical system. Moreover, ϕ is called a continuous random dynamical system if ϕ is continuous with respect to x for $t \geqslant 0$ and $\omega \in \Omega$.

Let $\mathcal{D}$ be the collection of all tempered random sets in X, and

$$\mathcal{D} = \{D = \{D(\omega)\}_{\omega\in\Omega} :\ D(\omega) \subseteq E_1,\ e^{-\varepsilon t} d(D(\theta_{-t}\omega)) \to 0,\ t \to +\infty\},$$

where $\gamma > 0$, $d(D(\theta_{-t}\omega)) = \sup_{u\in D(\theta_{-t}\omega)} \|u\|_E$.

Definition 2.2. A random set $A := \{A(\omega)\}_{\omega\in\Omega} \in X$ is called a random attractor for the random dynamical system ϕ if $P - a.s.$

(i) A is a random compact set, i.e. $A(\omega)$ is nonempty and compact for a.e. $\omega \in \Omega$ and $\omega \mapsto\ d(x, A(\omega))$ is measurable for every $x \in X$;
(ii) A is ϕ-invariant, i.e. $\phi(t, \omega, A(\omega)) = A(\theta_t\omega)$, for all $t \geqslant 0$ and a.e. $\omega \in \Omega$;
(iii) A attracts every set in X, *i.e.* for all bounded(and non-random) $B \subset X$,

$$\lim_{t\to\infty} d_H(\phi(t, \theta_{-t}\omega, B(\theta_{-t}\omega)), A(\omega)) = 0, \quad a.e.\ \omega \in \Omega,$$

where "$d_H(\cdot, \cdot)$" denotes the Hausdorff semi-distance between two subsets of X.

Theorem 2.3. *Let ϕ be a continuous random dynamical system on E over $(\Omega, F, \mathbb{P}, (\theta_t)_{t\in\mathbb{R}})$. Suppose that there exists a random compact set $K(\omega)$ such that for every bounded non-random set $B \in X$,*

$$\lim_{t\to\infty} d_H(\phi(t, \theta_{-t}\omega, B(\theta_{-t}\omega)), K(\omega)) = 0, \quad a.e.\ \omega \in \Omega.$$

Then the set

$$A = \{A(\omega)\}_{\omega\in\Omega} = \overline{\bigcup_{B\subset X} \Lambda_B(\omega)},$$

is a random attractor for ϕ, where the union is taken over all bounded $B \subset X$, and $\Lambda_B(\omega)$ is the ω-limits set of B given by

$$\Lambda_B(\omega) = \bigcap_{\tau\geqslant 0} \overline{\bigcup_{t\geqslant\tau} \phi(t, \theta_{-t}\omega, B(\theta_{-t}\omega))}, \quad \omega \in \Omega.$$

3 Existence and Uniqueness of Solutions

Denote $A = \triangle^2$, $A^{\frac{1}{2}} = -\triangle$ and $D(A) = \{u \in H^2(U) \cap H_0^1(U) \ |Au \in L^2(U)\}$. We can define $\mathcal{H}^r = D(A^{\frac{r}{4}})$. The space defined above is a Hilbert space with inner product and norm

$$(u,v)_r = (A^{\frac{r}{4}}u, A^{\frac{r}{4}}v),\ \|\cdot\|_r = \|A^{\frac{r}{4}}u\|,\ \ \forall\ u,v \in \mathcal{H}^r.$$

In particular, $D(A^0) = L^2(U)$, $D(A^{\frac{1}{2}}) = H^2(U) \cap H_0^1(U)$. The inner product and norm in $L^2(U)$ is denoted by $(\cdot,\cdot)$, $\|\cdot\|$, and in $H^2(U) \cap H_0^1(U)$ is denoted by $((\cdot,\cdot))$, $\|\cdot\|_2$. By (H_1), the space $\mathfrak{R}_{\mu,r} = L_\mu^2(\mathbb{R}^+, \mathcal{H}^r)$ is a Hilbert space with the inner product and norm, respectively

$$\begin{aligned}(\eta,\eta_1)_{\mu,r} &= \int_0^\infty \mu(s)(A^{\frac{r}{4}}\eta(s), A^{\frac{r}{4}}\eta_1(s))ds,\\ \|\eta\|_{\mu,r}^2 &= \int_0^\infty \mu(s)(A^{\frac{r}{4}}\eta(s), A^{\frac{r}{4}}\eta(s))ds,\end{aligned} \qquad \forall\ \eta,\ \eta_1 \in \mathcal{H}^r,$$

the linear operator $-\partial_s$ has domain

$$D(-\partial_s) = \{\eta \in H_\mu^1(\mathbb{R}^+, \mathcal{H}^r) : \eta(0) = 0\},$$

where $H_\mu^1(\mathbb{R}^+, \mathcal{H}^r) = \{\eta : \eta(s), \partial_s\eta(s) \in L_\mu^2(\mathbb{R}^+, \mathcal{H}^r)\}$.

To convert the problem (1.2) into a deterministic system with a random parameter, we consider Ornstein-Uhlenbeck equations

$$dz_j + z_j dt = dW_j(t),\ \ j = \{1, 2, \cdots, m\}, \tag{3.1}$$

and Ornstein-Uhlenbeck processes

$$z_j(\theta_t\omega_j) = -\int_{-\infty}^0 e^s(\theta_t\omega_j)(s)ds,\ \ t \in \mathbb{R}.$$

Put

$$z(\theta_t\omega) = z(x, \theta_t\omega) = \sum_{j=1}^m h_j z_j(\theta_t\omega_j), \tag{3.2}$$

which is a solution to

$$dz + zdt = \sum_{j=1}^m h_j dW_j.$$

Let

$$v(t,\omega,x) = u_t(t,\omega,x) + \varepsilon u(t,\omega,x),\ t \geq \tau, \quad \psi(t,\omega,x) = (u,v,\eta)^T,$$

where

$$\varepsilon = \min\{\frac{\lambda_1}{2+3\lambda_1}, \frac{4\kappa}{\delta}\}, \qquad \kappa = \|\mu\|_{L^2(\mathbb{R}^+)} > 0,$$

where $\lambda_1 > 0$ is the first eigenvalue of operator $\Delta^2 u$. The initial problem (1.2) is equivalent to the system in E :

$$\dot{\psi} + H(\psi) = N(Z,t,W(t)),\ \ \psi_\tau(\omega) = (u_0, u_1 + \varepsilon u_0, \eta_0)^T,\ t \geq \tau,\ \tau \in \mathbb{R}, \tag{3.3}$$

where

$$H(\psi) = \begin{pmatrix} \varepsilon u - v \\ (1-\varepsilon)Au + Av - \varepsilon^2 u - \varepsilon v + \int_0^\infty \mu(s)A\eta(s)ds \\ \varepsilon u - v + \eta_s \end{pmatrix} = -T_\varepsilon L T_\varepsilon(\psi),\ \ T_\varepsilon = \begin{pmatrix} 1 & 0 & 0 \\ \varepsilon & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.4}$$

Let

$$w(t,\omega,x)=u_t(t,\omega,x)+\varepsilon u(t,\omega,x)-z(\theta_t\omega),\quad \varphi=(u,w,\eta)^T,$$

then the problem (3.3) is equivalent to

$$\dot{\varphi}+H(\varphi)=F(\varphi,\theta_t\omega,t),\ \ \varphi_\tau(\omega)=(u_0,u_1+\varepsilon u_0-z(\theta_\tau\omega),\eta_0)^T,\ t\geq\tau,\ \tau\in\mathbb{R},\tag{3.5}$$

where

$$F(\varphi,\theta_t\omega,t)=\begin{pmatrix} z(\theta_t\omega)\\ -ku^+-(p-\|\nabla u\|^2)\Delta u+g(x)+\varepsilon z(\theta_t\omega)\\ z(\theta_t\omega)\end{pmatrix}.\tag{3.6}$$

By the standard theory of operators semigroup concerning the existence and uniqueness of solutions of evolution equations [20], we have:

Theorem 3.1. If $(H_1)-(H_2)$ hold. Then for each $\omega\in\Omega$ and $\varphi_\tau\in E$, there exists $T>0$, such that (3.5) has a unique mild solution $\varphi(\cdot,\omega,\psi_\tau)\in C([\tau,\tau+T);E)$ with $\varphi(\tau,\omega,\varphi_\tau)$ $=\varphi_\tau$, and

$$\varphi(t,\omega,\varphi_\tau)=e^{-H(t-\tau)}\varphi_\tau(\omega)+\int_\tau^t e^{-H(t-s)}F(s,\theta_s\omega,\varphi(s,\omega,\varphi_\tau))ds.\tag{3.7}$$

Furthermore, $\varphi(t,\omega,\varphi_\tau)$ is jointly continuous in φ_τ, and measurable in ω.

From Lemma 4.1 below and Theorem 3.1, the solution $\varphi(\cdot,\omega,\varphi_\tau)$ exists for $t\in[\tau,\infty)$. Thus the solution $\varphi(\cdot,\omega,\varphi_\tau)\in C([\tau,+\infty);E)$, and we can define a continuous random dynamical system over $\mathbb{R}$ and $(\Omega,F,\mathbb{P},(\theta_t)_{t\in\mathbb{R}})$:

$$\Phi(t,\omega):\varphi_\tau\mapsto\varphi(t,\omega,\varphi_\tau).\tag{3.8}$$

It is not hard to find

$$\Upsilon(t,\omega,Z_\tau)=R^{-1}_{\varepsilon,\theta_t\omega}\Phi(t,\omega)R_{\varepsilon,\theta_t\omega}:Z_\tau\to Z(t,\omega,Z_\tau)\tag{3.9}$$

and

$$\Psi(t,\omega,\psi_\tau)=T_\varepsilon\Upsilon T_{-\varepsilon}:\psi_\tau\to\psi(t,\omega,\psi_\tau)\tag{3.10}$$

are continuous random dynamical systems over $\mathbb{R}$ and $(\Omega,F,\mathbb{P},(\theta_t)_{t\in\mathbb{R}})$. So, Φ, Υ, and Ψ are equivalent to each other.

4 Random Absorbing Set

Lemma 4.1. Assume that $(H_1)-(H_2)$ hold. There exists a random variable $r_1(\omega)>0$ and a bounded ball $B(\omega)\in\mathcal{D}$, centered at 0 with random radius $r_1(\omega)$ such that for every $\{D(\omega)\}_{\omega\in\Omega}\in\mathcal{D}$, there exists $t_D(\omega)>0$ such that the solution $\varphi(t,\omega;\varphi_\tau(\omega))$ of (3.5) with initial value $(u_0,u_1+\varepsilon u_0,\eta_0)^T\in B$ satisfies, for P-$a.s.\omega\in\Omega$,

$$\begin{aligned}\|\varphi(\tau,\tau-t,\theta_{-\tau}\omega,\varphi_{\tau-t}(\theta_{-\tau}\omega))\|_E^2=&\|u(\tau,\tau-t,\theta_{-\tau}\omega,\varphi_{\tau-t}(\theta_{-\tau}\omega))\|_2^2\\&+\|w(\tau,\tau-t,\theta_{-\tau}\omega,\varphi_{\tau-t}(\theta_{-\tau}\omega))\|^2\\&+\|\eta(\tau,\tau-t,\theta_{-\tau}\omega,\varphi_{\tau-t}(\theta_{-\tau}\omega),s)\|_{\mu,2}^2\\ \leq& r_1^2(\omega),\quad\forall\ t\geq t_D(\omega),\end{aligned}\tag{4.1}$$

that is,

$$\varphi(t,\tau-t,\theta_{-t}\omega,B(\tau-t,\theta_{-t}\omega))\subseteq B(\omega),\quad\forall\ t\geq t_D(\omega).\tag{4.2}$$

Proof: Taking the inner product $(\cdot,\cdot)_E$ of (3.5) with $\varphi(r) = (u(r), w(r), \eta(r))^T$, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\|\varphi\|_E^2 + (H(\varphi),\varphi)_E \\ =&((z(\theta_t\omega), u)) - k(u^+, w) - ((p - \|\nabla u\|^2)\Delta u, w) + (g(x), w) \\ &+ \varepsilon(z(\theta_t\omega), w) + (z(\theta_t\omega), \eta)_{\mu,2}.\end{aligned} \tag{4.3}$$

Similar to the estimates of Lemma 4.1 in [21],

$$(H(\varphi),\varphi)_E \geq \frac{\varepsilon}{2}(\|u\|_2^2 + \|w\|^2) + \frac{\varepsilon}{4}\|u\|_2^2 + \frac{1}{2}\|w\|^2 + \frac{\delta}{4}\|\eta\|_{\mu,2}^2. \tag{4.4}$$

We estimate each term of the right-hand side of (4.3) as follows,

$$((z(\theta_t\omega), u)) \leq \frac{\varepsilon}{4}\|u\|_2^2 + \frac{1}{\varepsilon}|z(\theta_t\omega)|^2, \tag{4.5}$$

$$-k(u^+, w) \leq -\frac{1}{2}\frac{d}{dt}k\|u^+\|^2 - \frac{\varepsilon k}{2}\|u^+\|^2 + \frac{k}{2\varepsilon}|z(\theta_t\omega)|^2, \tag{4.6}$$

$$\begin{aligned}&-(p - \|u\|_1^2)(\Delta u, w) \\ =&-\frac{1}{4}\frac{d}{dt}(\|u\|_1^2 - p)^2 - \frac{\varepsilon}{2}(\|u\|_1^2 - p)^2 - \frac{\varepsilon}{2}\|u\|_1^4 + \frac{\varepsilon p^2}{2} - (\|u\|_1^2 - p)(\Delta u, z(\theta_t\omega)) \\ \leq&-\frac{1}{4}\frac{d}{dt}(\|u\|_1^2 - p)^2 - \frac{\varepsilon}{2}(\|u\|_1^2 - p)^2 - \frac{\varepsilon}{2}\|u\|_1^4 + \frac{\varepsilon p^2}{2} + \frac{\varepsilon}{4}(\|u\|_1^2 - p)^2 \\ &+ \frac{\varepsilon}{2}\|u\|_1^4 + \frac{1}{2\varepsilon^3}|z(\theta_t\omega)|^4 \\ =&-\frac{1}{4}\frac{d}{dt}(\|u\|_1^2 - p)^2 - \frac{\varepsilon}{4}(\|u\|_1^2 - p)^2 + \frac{\varepsilon p^2}{2} + \frac{1}{2\varepsilon^3}|z(\theta_t\omega)|^4,\end{aligned} \tag{4.7}$$

$$\varepsilon(z(\theta_t\omega), w)| \leq \frac{1}{4}\|w\|^2 + \varepsilon^2|z(\theta_t\omega)|^2, \tag{4.8}$$

$$(z(\theta_t\omega), \eta)_{\mu,2} \leq \frac{2\kappa}{\delta}\|z(\theta_t\omega)\|_2^2 + \frac{\delta}{8}\|\eta\|_{\mu,2}^2, \tag{4.9}$$

$$(g(x), w) \leq \frac{1}{4}\|w\|^2 + \|g\|^2. \tag{4.10}$$

Collecting all above (4.4) – (4.10), it yields

$$\begin{aligned}&\frac{d}{dt}\left(\|\varphi\|_E^2 + k\|u^+\|^2 + \frac{1}{2}(\|u\|_1^2 - p)^2\right) + \varepsilon\|u\|_2^2 + \varepsilon\|w\|^2 \\ &+ \frac{\delta}{4}\|\eta\|_{\mu,2}^2 + \varepsilon k\|u^+\|^2 + \frac{\varepsilon}{2}(\|u\|_1^2 - p)^2 \\ \leq& M(1 + |z(\theta_t\omega)|^2 + |z(\theta_t\omega)|^4),\end{aligned}$$

where $M = \max\{\frac{1}{\varepsilon} + \frac{k}{2\varepsilon} + \varepsilon^2 + \frac{2\kappa}{\delta}, \frac{1}{2\varepsilon^3}, \frac{\varepsilon p^2}{2} + \|g\|^2\}$. Choosing $\varepsilon_1 = \min\{\varepsilon, \frac{\delta}{4}\}$, applying Gronwall's inequality

$$\begin{aligned}&\|\varphi(t,\omega;\varphi(\tau,\omega))\|_E^2 \\ \leq& e^{-\varepsilon_1(t-\tau)}\left(\|\varphi(\tau,\omega)\|_E^2 + \frac{1}{2}(\|u_0\|_1^2 - p)^2 + k\|u_0^+\|^2\right) \\ &+ M\int_\tau^t e^{-\varepsilon_1(t-s)}(1 + |z(\theta_s\omega)|^2 + |z(\theta_s\omega)|^4)ds \\ \leq& 2e^{-\varepsilon_1(t-\tau)}\left(\|u_0\|_2^2 + \|u_1 + \varepsilon u_0\|^2 + \|\eta_0\|_{\mu,2}^2 + |z(\theta_\tau\omega)|^2 + (\|u_0\|_1^2 - p)^2 + k\|u_0^+\|^2\right) \\ &+ M\int_\tau^t e^{-\varepsilon_1(t-s)}(1 + |z(\theta_s\omega)|^2 + |z(\theta_s\omega)|^4)ds.\end{aligned} \tag{4.11}$$

Due to $\varphi(\tau,\omega)\in D(\omega)$ and $D(\omega)$ is tempered, it yields

$$\limsup_{t\to+\infty} e^{-\varepsilon_1(t-\tau)}\Big(\|u_0\|_2^2+\|u_1+\varepsilon u_0\|^2+\|\eta_0\|_{\mu,2}^2+|z(\theta_\tau\omega)|^2 + (\|u_0\|_1^2-p)^2+k\|u_0^+\|^2\Big) =0. \tag{4.12}$$

Take

$$r_1^2(\omega)=2M\int_{-\infty}^0 e^{-\varepsilon_1 t}(|z(\theta_s\omega)|^2+|z(\theta_s\omega)|^4)ds, \tag{4.13}$$

which is a tempered random variable, then by (4.11) and (4.12), the set $B(\omega)=\{\varphi\in E:\|\varphi\|_E\le r_1(\omega)\}$ is a closed measurable absorbing ball in $\mathcal{D}(E)$ and

$$\|\varphi(\tau,\tau-t,\theta_{-\tau}\omega,\varphi_{\tau-t}(\theta_{-\tau}\omega))\|_E^2\le r_1^2(\omega),\quad \forall\ t\ge t_D(\omega).$$

□

5 Decomposition of Solutions

Decomposing $\varphi=(u,w,\eta)^T$ of the system (3.5) into $\varphi=\varphi_L+\varphi_N$, then

$$\dot{\varphi}_L+H(\varphi_L)=0,\quad \varphi_L(\tau,\omega)=(u_0,u_1+\varepsilon u_0,\eta_0)^T,\ t\ge\tau,\ \tau\in\mathbb{R}, \tag{5.1}$$

and

$$\dot{\varphi_N}+H(\varphi_N)=\tilde{F}(\omega),\quad \varphi_N(\tau,\omega)=(0,-z(\omega),0)^T,\ t\ge\tau,\ \tau\in\mathbb{R}, \tag{5.2}$$

where

$$\tilde{F}(\omega)=\begin{pmatrix} z(\theta_t\omega)\\ -ku_N^+-(p-\|\nabla u_N\|^2)\Delta u_N+g(x)+\varepsilon z(\theta_t\omega)\\ z(\theta_t\omega)\end{pmatrix}. \tag{5.3}$$

For the solutions of Equations (5.1) and (5.2), we have the following estimate and regularity result, respectively.

Lemma 5.1. Suppose $D(\omega)$ be a bounded non-random subset of $\mathcal{D}$, for any $\varphi_L(\tau,\omega)=(u_0,u_1+\varepsilon u_0,\eta_0)^T\in D(\omega)$, there holds

$$\|\varphi_L(0,\omega;\varphi_L(\tau,\omega))\|_E^2\le r_2^2(\omega), \tag{5.4}$$

where $\varphi_L=(u_L,v_L,\eta_L)$ satisfies (5.1).

Proof: Taking the inner $(\cdot,\cdot)_E$ of (5.1) with $\varphi_L=(u_L,v_L,\eta_L)^T$, in which $v_L=u_{Lt}+\varepsilon u_L$, we get

$$\frac{1}{2}\frac{d}{dt}\|\varphi_L\|_E^2+(H(\varphi_L),\varphi_L)_E=0, \tag{5.5}$$

and

$$(H(\varphi_L),\varphi_L)_E\ge\frac{\varepsilon}{2}(\|u_L\|_2^2+\|v_L\|^2)+\frac{\varepsilon}{4}\|u_L\|_2^2+\frac{1}{2}\|v_L\|^2+\frac{\delta}{4}\|\eta_L\|_{\mu,2}^2, \tag{5.6}$$

Thus, putting (5.6) into (5.5), yields

$$\frac{d}{dt}\|\varphi_L\|_E^2+\varepsilon(\|u_L\|_2^2+\|v_L\|^2)+\frac{\varepsilon}{2}\|u_L\|_2^2+\|v_L\|^2+\frac{\delta}{2}\|\eta_L\|_{\mu,2}^2\le 0, \tag{5.7}$$

that is,

$$\frac{d}{dt}y_L+\varepsilon_2 y_L\le 0. \tag{5.8}$$

where $\varepsilon_2 = \min\{\varepsilon, \frac{\delta}{2}\}$ and

$$y_L = \|\varphi_L\|_E^2 \geq 0.$$

As $\varphi(0,\omega;\varphi(\tau,\omega)) = \varphi_L(0,\omega;\varphi_L(\tau,\omega)) + (0, z(\omega), 0) \in B(\omega)$, we obtain

$$\|\varphi_L(0,\omega,\varphi_L(\tau,\omega))\|_E \leq r_1(\omega) + |z(\omega)| = M_1(\omega).$$

By Gronwall's inequality,

$$\begin{aligned}\|\varphi_L(0,\omega,\varphi_L(\tau,\omega))\|_E^2 &\leq y_L(0,\omega,\varphi_L(\tau,\omega)) \\ &\leq y_L(\tau,\omega,\varphi_L(\tau,\omega))e^{\varepsilon_2\tau} \\ &\leq \|\varphi_L(\tau,\omega)\|_E^2 e^{\varepsilon_2\tau} \\ &\leq M_1^2(\omega)e^{\varepsilon_2\tau} = r_2^2(\omega).\end{aligned} \tag{5.9}$$

□

Lemma 5.2. Assume that $(H_1) - (H_2)$ hold, $p < \frac{\sqrt{\lambda_1}}{3}$, there exists a random radius $r_3(\omega)$, such that for P-$a.e.\omega \in \Omega$,

$$\|A^{\frac{1}{4}}u_N\|_2^2 + \|A^{\frac{1}{4}}u_{Nt}\|^2 + \|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2 \leq r_3^2(\omega). \tag{5.10}$$

Proof: Taking the inner of $(\cdot,\cdot)_E$ of (5.2) with $A^{\frac{1}{2}}\varphi_N = (A^{\frac{1}{2}}u_N, A^{\frac{1}{2}}w_N, A^{\frac{1}{2}}\eta_N)^T$, we get

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\Big(\|A^{\frac{1}{4}}u_N\|_2^2 + \|A^{\frac{1}{4}}w_N\|^2 + \|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2\Big) + (H(\varphi_N), A^{\frac{1}{2}}\varphi_N)_E \\ &= ((z(\theta_t\omega), A^{\frac{1}{2}}u_N)) - k(u_N^+, A^{\frac{1}{2}}w_N) - ((p - \|\nabla u_N\|^2)\Delta u_N, A^{\frac{1}{2}}w_N) \\ &+ (g(x), A^{\frac{1}{2}}w_N) + \varepsilon(z(\theta_t\omega), A^{\frac{1}{2}}w_N) + (z(\theta_t\omega), A^{\frac{1}{2}}\eta_N)_{\mu,2}.\end{aligned} \tag{5.11}$$

Following, we estimate the right terms in (5.11) as follows

$$\begin{aligned}&\Big(H(\varphi_N), A^{\frac{1}{2}}\varphi_N\Big)_E \\ \geq&\frac{\varepsilon}{2}\Big(\|A^{\frac{1}{4}}u_N\|_2^2 + \|A^{\frac{1}{4}}w_N\|^2\Big) + \frac{\varepsilon}{4}\|A^{\frac{1}{4}}u_N\|_2^2 + \frac{1}{2}\|A^{\frac{1}{4}}w_N\|^2 + \frac{\delta}{4}\|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2\end{aligned} \tag{5.12}$$

$$\Big((z(\theta_t\omega), A^{\frac{1}{2}}u_N)\Big) \leq \frac{\varepsilon}{4}\|A^{\frac{1}{4}}u_N\|_2^2 + \frac{1}{\varepsilon}|z(\theta_t\omega)|^2, \tag{5.13}$$

$$|-(ku_N^+, A^{\frac{1}{2}}w_N)| \leq 2k^2\|A^{\frac{1}{4}}u_N^+\|^2 + \frac{1}{8}\|A^{\frac{1}{4}}w_N\|^2, \tag{5.14}$$

$$\begin{aligned}&\Big((p - \|\nabla u_N\|^2)\Delta u_N, A^{\frac{1}{2}}w_N\Big) \\ =&\frac{1}{2}\frac{d}{dt}(\|u_N\|_1^2\|u_N\|_2^2 - p\|u_N\|_2^2) + \varepsilon(\|u_N\|_1^2\|u_N\|_2^2 - p\|u_N\|_2^2) \\ &+ (p - \|u_N\|_1^2)(A^{\frac{1}{2}}u_N, z(\theta_t\omega)) - \|u_N\|_2^3\|u_{N,t}\| \\ \geq&\frac{1}{2}\frac{d}{dt}(\|u_N\|_1^2\|u_N\|_2^2 - p\|u_N\|_2^2) + \frac{\varepsilon}{2}(\|u_N\|_1^2\|u_N\|_2^2 - p\|u_N\|_2^2) \\ &- \frac{\varepsilon}{2}|p|\|u_N\|_2^2 - (|p| + \|u_N\|_1^2)\|u_N\|_2|z(\theta_t\omega)| - \|u_N\|_2^3\|u_{N,t}\| \\ \geq&\frac{1}{2}\frac{d}{dt}(\|u_N\|_1^2\|u_N\|_2^2 - p\|u_N\|_2^2) + \frac{\varepsilon}{2}(\|u_N\|_1^2\|u_N\|_2^2 - p\|u_N\|_2^2) \\ &- \left(\frac{\varepsilon}{2}|p| + \frac{1}{2}(|p| + \|u_N\|_1^2)^2\right)\|u_N\|_2^2 - \frac{1}{2}|z(\theta_t\omega)|^2 - \|u_N\|_2^3\|u_{N,t}\|,\end{aligned} \tag{5.15}$$

$$(g(x), A^{\frac{1}{2}}w_N) \leq \|g\|_1^2 + \frac{1}{8}\|A^{\frac{1}{4}}w_N\|^2, \tag{5.16}$$

$$\varepsilon(z(\theta_t\omega), A^{\frac{1}{2}}w_N) \leq |z(\theta_t\omega)|^2 + \frac{1}{4}\|A^{\frac{1}{4}}w_N\|^2, \tag{5.17}$$

$$(z(\theta_t\omega), A^{\frac{1}{2}}\eta_N)_{\mu,2} \leq \frac{2\kappa}{\delta}|z(\theta_t\omega)|^2 + \frac{\delta}{8}\|A^{\frac{1}{4}}\eta_N\|_{\mu,2}, \tag{5.18}$$

By putting (5.12)-(5.18) into (5.11), yields

$$\begin{aligned}&\frac{d}{dt}\left(\|A^{\frac{1}{4}}u_N\|_2^2+\|A^{\frac{1}{4}}w_N\|^2+\|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2+\|u_N\|_1^2\|u_N\|_2^2-p\|u_N\|_2^2\right)\\&+\varepsilon_3\left(\|A^{\frac{1}{4}}u_N\|_2^2+\|A^{\frac{1}{4}}w_N\|^2+\|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2+\|u_N\|_1^2\|u_N\|_2^2-p\|u_N\|_2^2\right)\\\le&\left(\varepsilon|p|+(|p|+\|u_N\|_1^2)^2+\frac{4k^2}{\sqrt{\lambda_1}}\right)\|u_N\|_2^2+\left(3+\frac{2}{\varepsilon}+\frac{4\kappa}{\delta}\right)|z(\theta_t\omega)|^2\\&+2\|u_N\|_2^3\|u_{N,t}\|+\|g\|_1^2,\end{aligned}\tag{5.19}$$

that is

$$\begin{aligned}&\frac{d}{dt}y+\varepsilon_3 y\\\le&\left(\varepsilon|p|+(|p|+\|u_N\|_1^2)^2+\frac{4k^2}{\sqrt{\lambda_1}}\right)\|u_N\|_2^2\\&+\left(3+\frac{2}{\varepsilon}+\frac{4\kappa}{\delta}\right)|z(\theta_t\omega)|^2+2\|u_N\|_2^3\|u_{N,t}\|+\|g\|_1^2,\end{aligned}\tag{5.20}$$

where $\varepsilon_3=\min\{\varepsilon,\frac{\delta}{2}\}$ and

$$y=\|A^{\frac{1}{4}}u_N\|_2^2+\|A^{\frac{1}{4}}w_N\|^2+\|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2+\|u_N\|_1^2\|u_N\|_2^2-p\|u_N\|_2^2.$$

Since

$$y\ge C(p)(\|A^{\frac{1}{4}}u_N\|_2^2+\|A^{\frac{1}{4}}w_N\|^2+\|A^{\frac{1}{4}}\eta_N\|_{\mu,2}^2)>0,$$

here

$$C(p)=\begin{cases}1, & p\le 0,\\ 1-\frac{p}{\sqrt{\lambda_1}}, & 0<p<\frac{\sqrt{\lambda_1}}{3}.\end{cases}$$

From Lemma 4.1, we have

$$\begin{aligned}&\frac{d}{dt}y+\varepsilon_3 y\\\le&\left(\varepsilon|p|+(|p|+r_1^2(\omega))^2+\frac{4k^2}{\sqrt{\lambda_1}}\right)r_1^2(\omega)\\&+\left(3+\frac{2}{\varepsilon}+\frac{4\kappa}{\delta}\right)|z(\theta_t\omega)|^2+2r_1^4(\omega)+\|g\|_1^2,\end{aligned}\tag{5.21}$$

Taking advantage of Gronwall's lemma, it follows that

$$\begin{aligned}y\le& e^{-\varepsilon_3(t-\tau)}y_0+\int_\tau^t e^{-\varepsilon_3(t-s)}\left(\varepsilon|p|+(|p|+r_1^2(s,\omega))^2+\frac{4k^2}{\sqrt{\lambda_1}}\right)r_1^2(s,\omega)ds\\&+\left(3+\frac{2}{\varepsilon}+\frac{4\kappa}{\delta}\right)\int_\tau^t e^{-\varepsilon_3(t-s)}|z(\theta_s\omega)|^2ds+2\int_\tau^t e^{-\varepsilon_3(t-s)}r_1^4(s,\omega)ds+\frac{1}{\varepsilon}\|g\|_1^2.\end{aligned}\tag{5.22}$$

Let

$$\begin{aligned}r_3^2(\omega)=&\int_{-\infty}^0 e^{-\varepsilon_3 t}\left(\varepsilon|p|+(|p|+r_1^2(s,\omega))^2+\frac{4k^2}{\sqrt{\lambda_1}}\right)r_1^2(s,\omega)ds\\&+\left(3+\frac{2}{\varepsilon}+\frac{4\kappa}{\delta}\right)\int_{-\infty}^0 e^{-\varepsilon_3 t}|z(\theta_s\omega)|^2ds+2\int_{-\infty}^0 e^{-\varepsilon_3 t}r_1^4(s,\omega)ds+\frac{1}{\varepsilon}\|g\|_1^2.\end{aligned}$$

Since $z(\omega)$ is tempered, $\lim\limits_{t\to+\infty}e^{-\varepsilon_3 t}|z(\theta_{-t}\omega)|=0$ and $r_3^2(\omega)$ is a.s. bounded. □

6 Existence of Random Attractor

Lemma 6.1. [21, 13] Let X_0, X, X_1 be three *Banach* spaces such that $X_0 \hookrightarrow X \hookrightarrow X_1$, ,the first injection being compact. Let $Y \subset L^2_\mu(\mathbb{R}^+, X)$ satisfy the following hypotheses:

(*i*) Y is bounded in $L^2_\mu(\mathbb{R}^+, X_0) \cap H^1_\mu(\mathbb{R}^+, X_1)$,
(*ii*) $\sup_{\eta\in Y} \|\eta(s)\|^2_X \le K_0, \forall s \in \mathbb{R}^+$, for some $K_0 > 0$.

Then Y is relatively compact in $L^2_\mu(\mathbb{R}^+, X)$.

Note that for any $\forall\tau \in \mathbb{R}, \omega \in \Omega, t \ge 0$

$$\eta_N(t, \omega, \varphi(\tau, \omega), s) = \begin{cases} u_N(t, \omega, \varphi(\tau, \omega)) - u_N(t-s, \omega, \varphi(\tau, \omega)), & s \le t, \\ \eta_N(t, \omega, \varphi(\tau, \omega)), & s \ge t, \end{cases} \tag{6.1}$$

$$\eta_{Ns}(t, \omega, \varphi(\tau, \omega), s) = \begin{cases} u_{Nt}(t-s, \omega, \varphi(\tau, \omega)), & s \le t, \\ 0, & s \ge t. \end{cases} \tag{6.2}$$

Define

$$\tilde{B}(\tau, \omega) = \overline{\cup_{\varphi(\tau,\omega)\in B(r_1(\omega))} \cup_{t\ge 0} \eta_N(t, \omega, \varphi(\tau, \omega), s)},$$

where $\varphi = (u, w, \eta)^T$ is the solution of (3.5). By Lemma 5.2 and (6.1)-(6.2), we have

$$\max\{\|\eta_{Ns}(t, \omega, \varphi(\tau, \omega), s)\|^2_{\mu,1}, \|\eta_N(t, \omega, \varphi(\tau, \omega), s)\|^2_{\mu,3}\} \le 2r_3^2(\omega), \ \forall s \ge 0, \tag{6.3}$$

this shows $\tilde{B}(\tau, \omega)$ is bounded in $L^2_\mu(\mathbb{R}^+, \mathcal{H}^3) \cap H^1_\mu(\mathbb{R}^+, \mathcal{H}^1)$. Again, by Lemma 4.1, 5.2 and (6.1), the following

$$\sup_{\eta\in\tilde{B}(\tau,\omega), s\ge 0} \|\Delta\eta(s)\|^2 = \sup_{t\ge 0} \sup_{\varphi(\tau,\omega)\in B(r_1(\omega))} \|\Delta\eta_N(t, \omega, \varphi(\tau, \omega), s)\|^2 \le 2r_1^2(\omega) \tag{6.4}$$

is valid.

Therefore, by (H_1), for any $\eta \in \tilde{B}(\tau, \omega)$

$$\|\eta(s)\|^2_{\mu,2} = \int_0^\infty \mu(s)\|\eta(s)\|^2 ds \le 2r_1^1(\omega) \int_0^\infty \mu(s)e^{-\delta s} ds \le \frac{2r_1^2(\omega)}{\delta}. \tag{6.5}$$

We find that $\tilde{B}(\tau, \omega) \subset L^2_\mu(\mathbb{R}^+, H^2(U) \cap H^1_0(U))$ is bounded, together with Lemma 6.1,we get $\tilde{B}(\tau, \omega)$ is compact in $L^2_\mu(\mathbb{R}^+, H^2(U) \cap H^1_0(U))$. □

The main result for the random dynamical system Φ as follows:

Theorem 6.2. Suppose $(H1) - (H2)$ hold, $p < \frac{\sqrt{\lambda_1}}{3}$, then for any $\tau \in R, \omega \in \Omega$, the random dynamical system Φ associated with (3.5) possesses an attracting set $\Lambda(\tau, \omega) \subset E$, and a random attractor $\mathcal{A}(\tau, \omega) \subseteq \Lambda(\tau, \omega) \cap B(\omega)$.

Proof: For $\forall\tau \in R$, $\omega \in \Omega$,denote $B_1(\tau, \omega)$ be a closed ball in $H^3(U) \times H^1_0(U)$, its radius is $r_3(\omega)$. Suppose

$$\Lambda(\tau, \omega) = B_1(\tau, \omega) \times \tilde{B}(\tau, \omega), \tag{6.6}$$

then, $\Lambda(\tau, \omega) \in \mathcal{D}$. Since $H^3(U) \times H^1_0(U) \hookrightarrow (H^2(U) \cap H^1_0(U)) \times L^2(U)$, $B_1(\tau, \omega) \hookrightarrow (H^2(U) \cap H^1_0(U)) \times L^2(U)$. Additional, $\tilde{B}(\tau, \omega)$ is compact in $\mathfrak{R}_{\mu,2}$, so $\Lambda(\tau, \omega)$ is compact in E. Now we show that $\Lambda(\tau, \omega)$ is an attracting set.

For every $B(\tau,\omega) \in \mathcal{D}$,

$$\lim_{t\to\infty} d_H(\Phi(t,\tau-t,\theta_{-t}\omega, B(\tau-t,\theta_{-t}\omega)),\Lambda(\tau,\omega)) = 0. \tag{6.7}$$

By Lemma 5.1, we have

$$\varphi_N(0,\omega,\varphi(\tau,\omega)) =\varphi(0,\omega,\varphi(\tau,\omega)) - \varphi_L(0,\omega,\varphi_L(\tau,\omega)) \in \Lambda(\tau,\omega). \tag{6.8}$$

Thus, by Lemma 6.1, we have

$$\inf_{\psi\in\Lambda(\tau,\omega)} \|\varphi(0,\omega,\varphi(\tau,\omega)) - \psi\|_E^2 \le \|\varphi_L(0,\omega,\varphi_L(\tau,\omega))\|_E^2 \le M_1^2(\omega)e^{\varepsilon_2\tau}, \tau \le 0. \tag{6.9}$$

For $\forall t > 0$

$$d_H\left(\Phi(t,\tau-t,\theta_{-t}\omega, B(\tau-t,\theta_{-t}\omega)),\Lambda(\tau,\omega)\right) \le M_1^2(\omega)e^{-\varepsilon_2 t}. \tag{6.10}$$

Ultimately, from the relation between Φ and Ψ, we know that for any non-random bounded set $B \subset E$ P-a.s.

$$d_H(\Psi(t,\tau-t,\theta_{-t}\omega, B(\tau-t,\theta_{-t}\omega)),\Lambda(\tau,\omega)) \to 0, \ t\to+\infty. \quad P-a.s. \tag{6.11}$$

Hence, by Lemma 4.1 and Theorem 2.3, the random dynamical system Φ associated with (3.5) possesses a random attractor $\mathcal{A}(\tau,\omega) \subseteq \Lambda(\tau,\omega) \cap B(\omega)$. □

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Competing Interests

Author has declared that no competing interests exist.

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