

A Comparative Study of Fourier Series Models and Seasonal - Autoregressive Integrated Moving Average Model of Rainfall Data in Port Harcourt

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

This study compares the Seasonal autoregressive integrated moving average (SARIMA) model within Fourier time series model in modelling rainfall data in Port Harcourt Rivers State from 2000-2014. The time plot of the series showed Seasonality but a not obvious trend. The raw data is nonstationary at the level. Time plot of the seasonal differencing of rainfall at lag12 showed a stationary process with seasonality at lag 12 on the PACF and ACF of the series. The periodogram plot reveals that there exist both short and long term cycles within the period. The Fourier series and the seasonal autoregressive moving average models are reduced to 12month of seasonal component.(s = q = 12) The Akaike Information Criterion (AIC) was used to select better models. The best model is the model that minimises the information criterion. It was observed that SARIMA (1,0,1)(1,1,1)₁₂ models have a minimum AIC value. Hence, SARIMA model performs better in modelling the rainfall data in Port Harcourt then the Fourier series models.

Keywords: Rainfall in Port Harcourt; periodogram; seasonal ARIMA model; fourier series models.

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1 Introduction

In recent time, the incidence of unusual weather patterns as they affect both wet and dry season has been observing in Nigeria. Sometimes heavier rainfall may occur and the rain is prolonged and extends into the dry season. Sometimes the August break may or may not even occur in some years in Port Harcourt [1]. The Rainy season in Port Harcourt is between March and October through November. The peaks of the wet season are in June and July, and some years extent to august and September. Why the only dry months are December and January. However, the rainfall in February varies with little or no rainfall. Moreover, Rainfall is one of the most important natural factors that determine agricultural production in and across the globe, particularly in Port Harcourt, Rivers State. The inconsistency of rainfall and the pattern of high or low rainfall is very significant for agriculture as well as the economy of Rivers State [2]. However, studying rainfall pattern is a critical process and have serious implications in climate change, agriculture and natural hazards such as floods, earthquake etc Consequently, Port Harcourt is the capital of Rivers State. It is surrounded by other major states such as AKWA IBOM STATE, IMO STATE, ABIA STATE AND BAYELSA STATE. In July, half of the year humidity is over 80% and the average monthly rainfall of 334.9 but the humidity is higher in September with average rainfall 360.4667. The vegetation is evergreen and luxuriant due to heavy rainfall. The rainfall series are usually seasonal and this pattern can better be model using a stochastics non-linear modelling method that characterises the behaviour of the series. Seasonality is one of the most important concepts in time series analysis [3].

2 Literature Review

Omkara CO et al. [4] Studies the application of Fourier series analysis to seasonal Fourier time series model that can lead to reliable forecast. However, the study estimates data for the expectations of time series analysis; apply the necessary conversion to the data and fits multiplicative and additive Fourier series analysis models. The result shows the square transformation which outperforms the others is adopted. Consequently, each of the multiplicative and additive Fourier Series Analysis models fitted to the transformed data. Then, their subject the series to a test for white noise based on spectral analysis. The result of this test shows that only the multiplicative model is adequate. Hence, it used to make a forecast of the future [5] Examined the reversion processed with periodic function using Fourier series analysis. There describes the procedure to estimate the parameters in mean reversion processes with functional tendency defined by a periodic continuous deterministic function, expressed as a series of truncated Fourier series. Two phases of estimation are defined, in the first phase through Gaussian techniques using the Euler-Maruyama discretization, the maximum likelihood function, the estimators of the external parameters and an estimation of the expected value of the process. In the second phase [6]. Investigate the seasonal and linear trend models on monthly rainfall data in Calabar. He aims at modelling a periodic time series function with a linear trend. A Fourier series representation with the detrended linear function was proposed. He express the series as a combination of linear trend component and a linear combination of trigonometric functions. The method was applied to rainfall data and the proposed model was found to give well-fitted models. A comparative study was carried out with the complete Fourier representation [7]. Compared autoregressive integrated moving average model, Fourier time series model and the wavelet model. The variable used for the study is the consumer price index (CPI) data. The Autoregressive integrated moving average and Fourier model was obtained by combining both the linear and fluctuation components of the series. The three models were subjected to comparative analysis tests. The wavelet model performs better than the Autoregressive integrated moving and Fourier time series model [8] Compares seasonal autoregressive integrated moving average model with Fourier time series model in modelling Rainfall data in Akwa Ibom State. The results reveal that Fourier time series model best fit the results than the SARIMA model base on Akaike Information Criterion (AIC). The study recommends Fourier time series models for modelling and forecasting Akwa Ibom State rainfall data [3]. Examined the Nigeria inflation rate using seasonal ARIMA mode from 2003-2011. In their discussion, the time plot showed a secular movement and the seasonal differencing showed a seasonality but the direction of the movement is not clean. The non-seasonal differencing of series produced a stationary series. The plot of the ACF shows a spike at lag 12 showing seasonal MA component. The plot of the PACF showed no spike at the beginning suggesting a non -seasonal

MA component. The adequate model for the inflation rate in Nigeria follow a SARIMA (0,1,1)*(0,1,1)₁₂ model [9]. Studied monthly Nigeria Treasury bill rates by Box-Jenkins techniques from January 2006 to December 2014. The time plot of data showed a downward movement from 2006 to 2009 and upward movement to 2013. The 12-month seasonal differencing produce a horizontal trend and seasonality are not clear. The test for stationarity showed that the monthly Treasury bill rate is stationary. The plot of the ACF shows a negative spike at lag 12, this indicates seasonality. The acceptable model for Treasury bill rates is SARIMA (011)*(011)₁₂ model [2]. Studied rainfall pattern in Ghana as a seasonal ARIMA process between 1974 to 2010 using the Box- Jenkins method. The time plot shows seasonality and trend. The test statistics showed that the series is non-stationary. The twelfth month seasonal differencing make the series stationary. The ACF and PACF showed a spike at lag 12 which indicate seasonality. Four models were estimated for the series but the best model was chosen base on the least BIC, which is estimated as SARIMA (0,0,0)*(2,1,0)₁₂.

Ekpenyong EJ et al. [10] Study the monthly Rainfall data using seasonal autoregressive moving average. The time plot should a clear evidences of seasonal movement the data, a seasonal movement of 12 is notice from the graph. Their apply a seasonal differencing of lag 12 to the series. The plot of the correlogram indicate an autoregressive models of order 5. The best models in modelling the series is SARIMA ((5, 1, 0)(0, 1, 1)₁₂) [11]. Studies the movement of rainfall in Mahanadi River Basin in India using seasonal autoregressive integrated moving average. In their research, the ACF and the PACF plot should a spike at lag one indicating autoregressive of order one and moving average of order one. The series was stationary at order one. The AKaike information criterion (AIC), goodness of fit (Chi-square), R² (coefficient of determination), mean square error and mea absolute error were used to determine the adequate of the models. The best model was used for 12 years forest [12]. Examined the temperature and rainfall on a seasoan movement used seasonal autoregressive models approached to forest the feature rainfall and temperature of Mirzapur and Uttar Pradesh in India. Their concluded that the seasonal autoregressive integrated moving average provide reliable and satisfactory forest on a monthly scale.

3 Methodology

In this method, a periodic time series is first observed whether it contains a linear trend or seasonal in trend. Visual inspection of the time plot of the original data can reveal this pattern. Assuming a linear trend is detected, a linear regression model of the form

$$Y_t = \beta_0 + \beta_1 t + e_t \tag{1}$$

is first fitted to the data;

where Y_t is the observed time series, t is the time points ($t = 1, 2, \dots, n$), n is the number of observations, β_0 and β_1 are the regression parameters, and e_t is the error component [13].

3.1 Seasonal autoregressive integrated moving average model of order $(p, d, q) \times (P, D, Q)_s$

If Seasonality is detected, a seasonal autoregressive integrated moving average is needed in modelling the linear component of the models. The seasonal ARIMA model incorporates both non-seasonal and seasonal factors in a multiplicative model. One shorthand notation for the model is;

$$\text{SARIMA } (p, d, q) \times (P, D, Q)_s \tag{2}$$

Where p = non-seasonal AR of order p , d = non-seasonal differencing, q = non-seasonal MA order of q , P = seasonal AR order P , D = seasonal differencing, Q = seasonal MA order Q , and S = time

span of repeating seasonal pattern [14]. Without differencing operations, the model can be represented as follow,

$$\Phi(B^s)\varphi(B)(y_t - \mu) = \Theta(B^s x)\theta(B)\varepsilon_t \quad (3)$$

The non-seasonal components are:

$$\text{AR: } \varphi(B) = 1 - \varphi_{1B^1} - \dots - \varphi_{pB^p} \quad (4)$$

$$\text{MA: } \theta(B) = 1 + \theta_{1B^1} + \dots + \theta_{qB^q} \quad (5)$$

The seasonal components are: Seasonal

$$\text{AR: } \Phi(B^s) = 1 - \varphi_{1B^s} - \dots - \varphi_{pB^{ps}} \quad (6)$$

$$\text{Seasonal MA: } \Theta(B^s) = 1 + \theta_{1B^s} + \dots + \theta_{qB^{sq}} \quad (7)$$

Seasonal differencing is defined as a difference between a value and a value with lag that is a multiple of S . When $S = 12$, which may occur with monthly data, a seasonal difference is

$$(1 - B_{12})y_t = y_t - y_{t-12} \quad (8)$$

The differences (from the previous year) may be about the same for each month of the year giving us a stationary series. Seasonal differencing removes the seasonal trend and can also get rid of a seasonal random walk type of non-stationary [15].

3.2 Fourier Series Representation of the seasonal Series y_t

Given a time series of n observations, the Fourier representation is

$$y_t = \sum_{i=1}^q (\alpha_i \cos 2\pi f_i t + \beta_i \sin 2\pi f_i t) + e_t \quad (9)$$

estimated by

$$\hat{y}_t = \sum_{i=1}^q (a_i \cos 2\pi f_i t + b_i \sin 2\pi f_i t) \quad (10)$$

where $= n/2$, $a_i = \frac{2}{n} \sum_{t=1}^n y_t \cos 2\pi f_i t$, $b_i = \frac{2}{n} \sum_{t=1}^n y_t \sin 2\pi f_i t$,

$e_t \sim NIID(0, \sigma^2)$; period $= p_i = n/i$ and $f_i = i/n$ is the i^{th} harmonic of the fundamental frequency $1/n$ [16].

3.3 The Peridogram

The periodogram is defined as the function of intensities $I(f_i)$ at frequency $f_i = i/n$ and is given as

$$I(f_i) = \frac{n}{2} (a_i^2 + b_i^2) \quad (11)$$

$i = 1, 2, \dots, q$.

The periodogram is the plot of the intensities against the frequencies or periods. The periodogram $I(f_i)$ is simply the sum of squares associated with the pair of coefficients (a_i, b_i) and hence with the frequency f_i or period p_i . That is,

$$\sum_{t=1}^q (y_t - \bar{y})^2 = \sum_{t=1}^{n/2} I(f_i). \quad (12)$$

In the context at hand, the periodogram is used to determine the seasonality or periodicity of a time series. This is usually indicated by the largest peak in the periodogram plot [8].

3.4 The Spectrum

The sample spectrum is obtained by allowing the frequency f to vary continuously in the range 0 to 0.5 cycle so that the periodogram can be re-defined as

$$I(f) = \frac{n}{2} (a_f^2 + b_f^2) \quad (13)$$

; $0 \leq f \leq 0.5$.

The function $I(f)$ is called the spectrum.

3.5 Spectral density function

Spectral density is the Fourier transform of the autocorrelation function and is estimated by

$$g(f) = 2[1 + \sum_{k=1}^{\infty} \rho_k \cos(2\pi f k)] \quad (14)$$

$0 \leq f \leq 0.5$

where ρ_k is the autocorrelation at lag k . The spectral density performs the same function as the periodogram. The period or seasonality of a time series is obtained at where the spectral density is maximum.

3.6 The seasonal fourier representation

Rather than fitting the entire Fourier series expression in equation (7), we fit only the Fourier terms up to the season detected by the periodogram. That is, suppose the seasonal component is determined by the periodogram, series y_t is $s = q$, seasonal then equation (7) reduces to $y_t = \sum_{i=1}^s (\alpha_i \cos 2\pi f_i t + \beta_i \sin 2\pi f_i t) + \varepsilon_t$ (15)

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 t + \sum_{i=1}^s (\alpha_i \cos 2\pi f_i t + b_i \sin 2\pi f_i t) \quad (16)$$

The expression is less cumbersome in carrying out analysis than the complete Fourier form expressed in equation (7). The model (15) can be fitted to any periodic or seasonal data and the estimated residuals $\hat{\varepsilon}_t$ obtained from (15) can be tested for white noise to determine whether the model is adequate or not [6].

4 Data Analysis and Result

The data used for this work is the average monthly rainfall data (Y_t) in Port Harcourt Rivers State, Nigeria between 2000-2014 (Nigeria metrological Agencies); and the analysis is carried out using Minitab and gretl software.

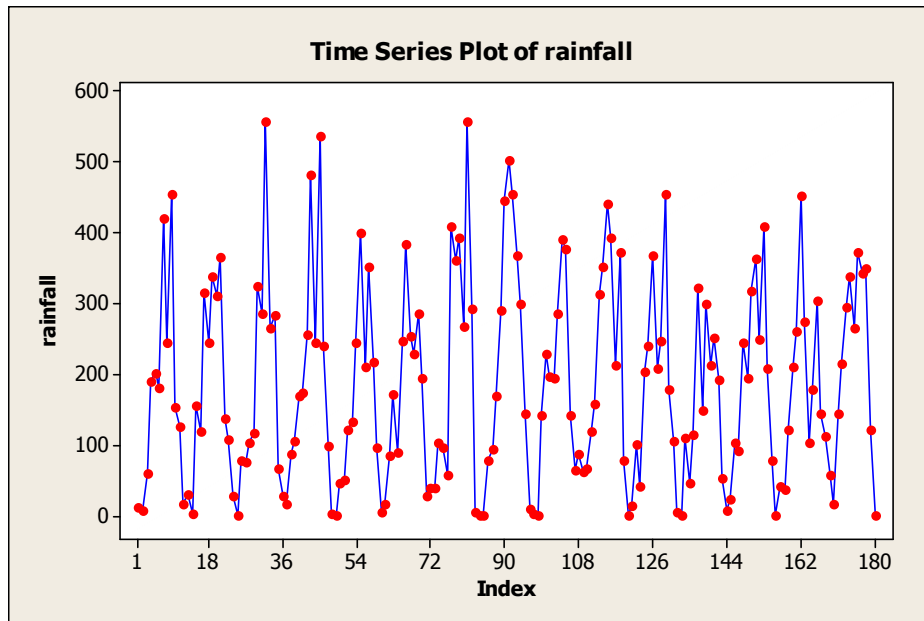


Fig. 1. Time plot of original series of rainfall data

The time plot of the original series shows many data points since rain is a seasonal phenomenon the data show seasonality but a not obvious trend. The present of Seasonality in any series can result in spurious regression, and cause the series is not stationary (stationary in the series, means constant mean and variance).

Periodogram Plot

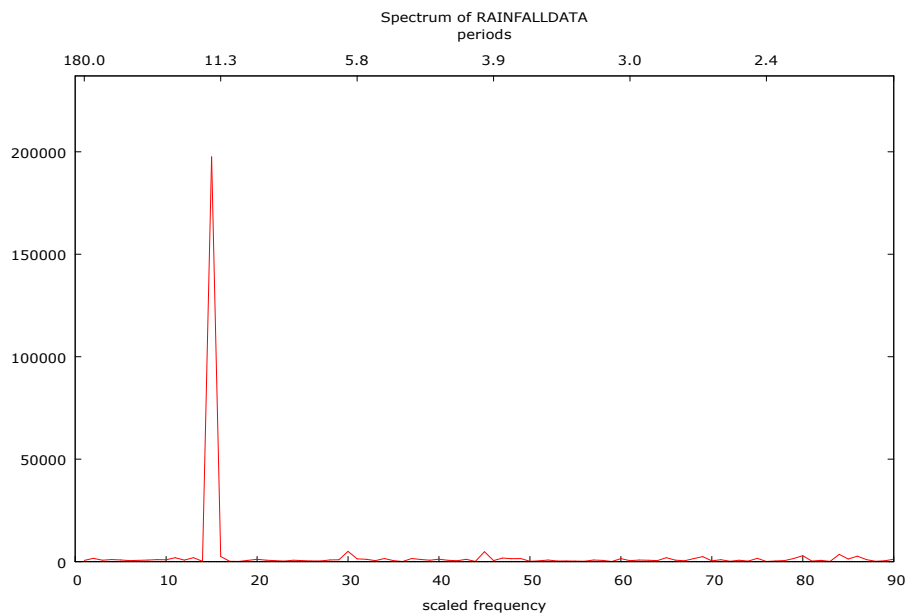


Fig. 2. Periodogram plot

The periodogram is used to measure the fluctuation pattern that occur in the series and to identify the frequencies of a time series. The plot of the period against the frequency reveals that, there exist about five cycles within the period under consideration. The longest cycle is for fifteen (15) month; this is known by checking the largest spectrum density $[I(f_i)]$ respectively in periodogram. The periodogram analysis was conducted using the Gretl software. From the plot, it is observed that the frequency corresponding to period is when $s = \frac{N}{i} = \frac{180}{15} = 12$. This indicate that it takes 12 periods or season to complete cycle. The Fourier series and the seasonal Autoregressive moving average models are reduced 12-month seasonal component. ($s = q = 12$) [6].

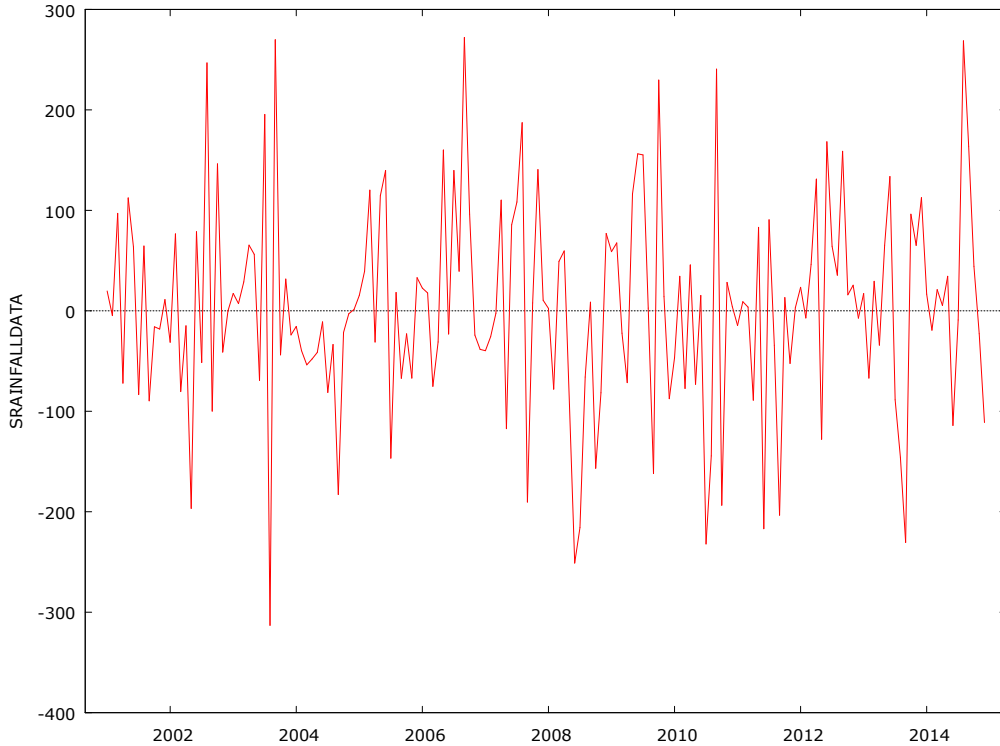


Fig. 3. Time plot of seasonal differencing of rainfall data at Lag 12

Table 1. Estimation of sarima(1.0.1)(1.1.1)₁₂

Models	co. off	se.coeff	T	p. Value
AR(1)	-0.2253	0.2055	-1.1	0.282
SAR(12)	-1.016	0.0928	-10.94	0.000
MA(1)	0.9263	0.1109	8.35	0.00
SMA(12)	0.8267	0.2364	3.5	0.001

The estimated SARIMA models for the series of rainfall is shows below

$$Y_t = \varepsilon_t - 1.016Y_{t-12} - 0.2258Y_{t-1} - 0.24Y_{t-13} + 0.8267\varepsilon_{t-12} - 0.9262\varepsilon_{t-1} - 0.7656\varepsilon_{t-13}$$

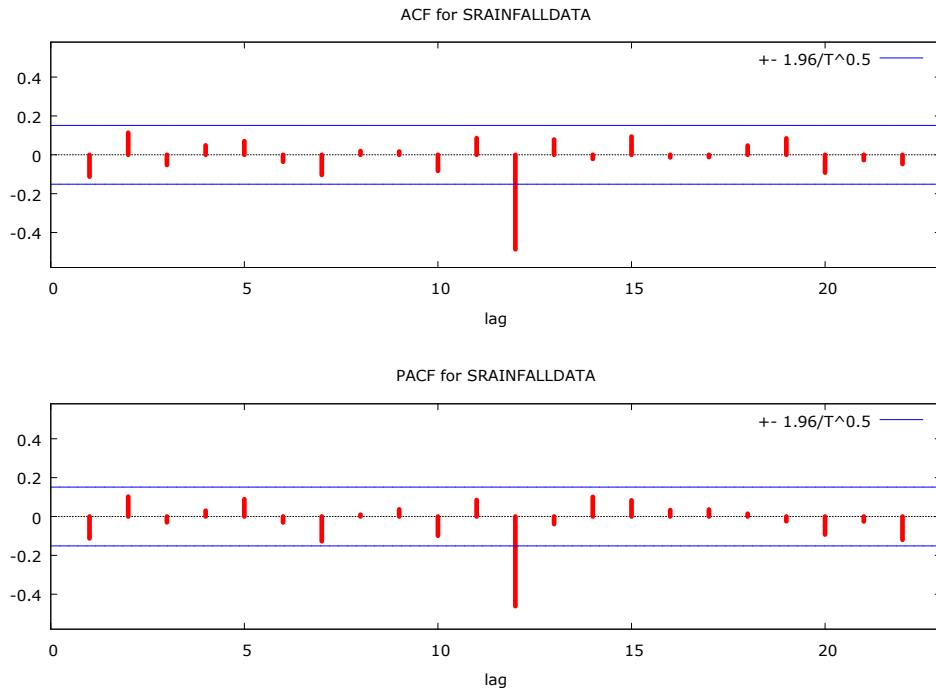


Fig. 4. The correlogram of seasonal differencing of rainfall data at lag 12

The correlogram of seasonal differencing data is in Fig. (2), showing that the series is seasonal with a spike at lag 12 in the ACF and PACF and a tapering pattern on the ACF and PACF showing seasonal behaviour.

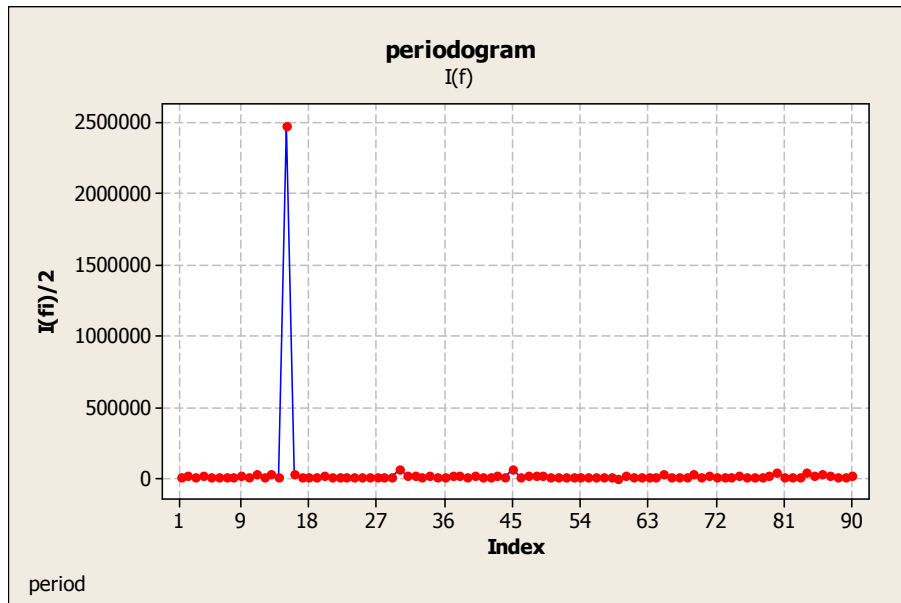


Fig. 5. Plot of intensities against frequency

Table 2. Parameters estimates from fourier model

Predictor	Coef	SE Coef	t	p	VIF
constant	183.69	10.72	17.14	0.000	0
Cos(W1t)	-0.05583	0.0442	-0.87	0.387	1.076
Sin(W1t)	0.04166	0.06905	0.6	0.547	1.071
Cos(W2t)	0.09001	0.065550	1.37	0.172	1.089
Sin(W2t)	0.05545	0.06771	0.82	0.414	1.055
Cos(W3t)	-0.00130	0.0833	-0.02	0.988	1.760
Sin(W3t)	-0.0318	0.08356	-0.38	0.708	1.609
Cos(W4t)	0.04661	0.08451	0.55	0.582	1.777
Sin(W4t)	-0.03601	0.08248	-0.44	0.663	1.599
Cos(W5t)	-0.08149	0.08367	-0.97	0.332	1.627
Sin(W5t)	0.10986	0.08367	1.31	0.191	1.761
Cos(W6t)	0.04391	0.08284	0.53	0.597	1.755
Sin(W6t)	0.03305	0.08454	0.39	0.696	1.632
Cos(W7t)	0.00017	0.08700	0.00	0.998	1.813
Sin(W7t)	0.04953	0.08099	0.61	0.542	1.603
Cos(W8t)	0.03721	0.08136	0.46	0.648	1.751
Sin(W8t)	-0.07863	0.08621	-0.91	0.363	1.631
Cos(W9t)	-0.00682	0.08328	-0.08	0.935	1.702
Sin(W9t)	-0.06256	0.08410	-0.74	0.458	1.687
Cos(W10t)	-0.07058	0.08453	-0.84	0.405	1.849
Sin(W10t)	-0.02479	0.08372	-0.30	0.768	1.578
Cos(W11t)	0.06179	0.08157	0.76	0.451	1.688
Sin(W11t)	-0.08198	0.08574	-0.96	0.340	1.698
Cos(W12t)	0.07409	0.08228	0.90	0.369	1.651
Sin(W12t)	0.02331	0.08511	0.27	0.785	1.739

5 Fitting the General Fourier Series Model

$$\begin{aligned}
 Y_t = & 184 - 0.0558 X_t \cos(W1t) + 0.0417 X_t \sin(W1t) + 0.0900 X_t \cos(W2t) + 0.0555 X_t \sin(W2t) - 0.0013 \\
 & X_t \cos(W3t) - 0.0314 X_t \sin(W3t) + 0.0466 X_t \cos(W4t) - 0.0360 X_t \sin(W4t) - 0.0815 X_t \cos(W5t) + 0.110 \\
 & X_t \sin(W5t) + 0.0439 X_t \cos(W6t) + 0.0330 X_t \sin(W6t) + 0.0002 X_t \cos(W7t) + 0.0495 X_t \sin(W7t) + \\
 & 0.0372 X_t \cos(W8t) - 0.0786 X_t \sin(W8t) - 0.0068 X_t \cos(W9t) - 0.0626 X_t \sin(W9t) - 0.0706 X_t \cos(W10t) - \\
 & 0.0248 X_t \sin(W10t) + 0.0618 X_t \cos(W11t) - 0.0820 X_t \sin(W11t) + 0.0741 X_t \cos(W12t) + 0.0233 \\
 & X_t \sin(W12t)
 \end{aligned}
 \tag{20}$$

5.1 Models selection criterion

The Akaike information criterion is used to select the better models. The best model is the model that minimises the information criterion. The formula is giving below

$$AIC = \frac{2K}{n} + \ln\left(\frac{RSS}{N}\right)
 \tag{21}$$

Where K is the number of the parameter in the models, RSS is the residual sum of square n is the number of Observation in the error of the models [9].

Table 3. Models selection criterion

Models	AIC
SARIMA(1,0,1)(11,1) ₁₂	11.6452
Fourier series models	12.6027

From Table3, it observed that SARIMA models have a lower AIC value. Hence, SARIMA model performs better in modelling the rainfall data Port Harcourt then the Fourier series models [17].

6 Discussion

The plot of the original series reveals a seasonal movement in the rainfall data in Port Harcourt. The present of seasonality can cause spurious regression, we need to model series using seasonal autoregressive integrated moving average and seasonal Fourier series models. The periodogram plot reveals that there exist both short and long term cycles within the period. The long term cycle is within 5 months while the short term cycle is within 20 months. This is revealed in the plot of the periodogram against the frequency. The need to models data using two different models was motivated to compare the performance of SARIMA and Fourier time series models. The present of seasonality was also revealed in the plot of the partial autocorrelation function and its autocorrelation function. The seasonal differencing component is at lag $s = 12$, whose model was obtained as SARIMA (1, 0, 1) (1 1, 1)₁₂. We can also confirm it from the plots of ACF and PACF. The Fourier series model, multiple regression analysis (for $i = 1, 2, \dots, 12$) was carried out, and the estimates are in Table 2. The estimates in Table 2 are from regression analysis after dropping the insignificant parameters from the earlier regression analysis. The choice of the best models was based on Akaike information criterion (AIC). The SARIMA model has minimum information criterion over Fourier series model in the analysis of rainfall data in Port Harcourt.

7 Conclusion

This research work is based on rainfall data in Port Harcourt from 2000 to 2014. Two models technique was adapted in modelling the seasonal movement in the series. (seasonal autoregressive integrated moving average and Fourier series analysis). The Akaike information criterion (AIC) was used to compare the best models. The researcher conclude that SARIMA (1, 0, 1) (1 1, 1)₁₂. Is appropriate in modelling seasonal movement of rainfall data in Port Harcourt Base on Akaike information criterion (AIC) of (11.6452). Since the best model is seasonal model the farmer in the study area should note that the farming period is in January and the harvesting period is in December

Competing Interests

Authors have declared that no competing interests exist.

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