

Wiener Index of Fuzzy Graph by Using Strong Domination

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Authors' contributions

This work was carried out in collaboration between both authors. Author SHAE designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author MMQS managed the analyses of the study. Author MMQS the literature searches. Both authors read and approved the final manuscript.

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Abstract

Aims/ Objectives: This paper presents a new method to calculate the Wiener index of a fuzzy graph by using strong domination number γ_s of a fuzzy graph G . This method is more useful than other methods because it saves time and efforts and doesn't require more calculations, if the edges number is very larg (n). The Wiener index of some standard fuzzy graphs are investigated. At last, we find the relationship between strong domination number γ_s and the average $\mu(G)$ of a fuzzy graph G was studied with suitable examples.

Keywords: Fuzzy graph; strong domination; Wiener index.

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1 Introduction

Fuzzy graph is one of the application tool in the field of mathematics, which allow the users to describe the relationship between any notions easily, because the nature of fuzziness is favorable for environment. Harold Wiener suggested the first definition of the Wiener index $W(G)$ as "The sum of the distance between all the pairs vertices of a graph G " in (1947)[1, 2]. Fuzzy graphs are beneficial to give more accurate and flexible system as compared to the classical models. A topological index is a numerical quantity for the structural graph of a molecule. Generally the topological indices are familiar in chemistry but a graph structure is of the mathematical background and Harold, who developed it in his theory and constructed a huge branch of chemical graph theory. Wiener usage indices to find the properties of the type of alkanes known as Paraffin-Moreover, it isn't only an application in the field of chemistry, but it can be applied in all areas including computer science, networking human traffic and internet routing. Perhaps the fastest growing area within graph and fuzzy graph is the study of domination, the reason being its many and varied applications in such fields. There are several types of domination depending upon the nature of domination like strong domination in fuzzy graph, which motivated us to find the Wiener index due to the work of domination on fuzzy graph and the possibility of using it. Using the normal methods to calculate the Wiener index is difficult to find if the number of edges is very large, so it is more easier to save time and efforts. In (2015) [3] the concept of strong dominating set introduced and investigated by O.T. Maniusha and M.S. sunitha.

2 Preliminaries

This section is focus on the basic definitions results related to fuzzy graph, strong domination in fuzzy graph and Wiener index.

A fuzzy graph $G = (\mu, \rho)$ where μ is a fuzzy subset of V and ρ is a fuzzy relation on μ such that $\rho(u, v) \leq \mu(u) \wedge \mu(v); \forall u, v \in V$. We assume that V is finite and non empty, ρ is reflexive and symmetric. For all $u, v \in V$. The order p and size q of a fuzzy graph G are defined as $p = \sum_{v \in V(G)} \mu(v)$ and $q = \sum_{u, v \in V(G)} \rho(uv)$.

The complement of a fuzzy graph G , denoted by \bar{G} is defined as $\bar{G} = (\mu, \bar{\rho})$ where $\bar{\rho}(uv) = \mu(u) \wedge \mu(v) - \rho(uv)$, for all $u, v \in V$. A weakest edge of G is an edge with least membership value. A path P of length n is a sequence of distinct vertices u_0, u_1, \dots, u_n such that $\mu(u_i, u_{i+1}) > 0, i = 1, 2, 3, \dots, n$ and the degree of membership of a weakest edge in the Path is defined its strength, if $u_0 = u_n$ and $n \geq 3$, then P is called a cycle and a cycle P is called a fuzzy cycle if it contains more than one weakest edge. A fuzzy graph $G = (\mu, \rho)$ is a complete fuzzy graph if $\rho(u, v) = \mu(u) \wedge \mu(v), \forall u, v \in \mu^*$.

The strength of connectedness between two vertices u and v is defined as the maximum of the strength of all paths between u and v and is denoted by $CONN_G(u, v)$. A $u - v$ path P is called a strongest $u - v$ path if its strength equals $CONN_G(u, v)$. A fuzzy graph G is connected if for every $u, v \in \mu^*, CONN_G(u, v) > 0$ throughout this, we assume that G is connected. An edge of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end vertices when it is deleted, depending $CONN_G(u, v)$ of an edge uv in fuzzy graph G . Thus an edge uv is a strong edge if it is either α -strong or β -strong [4]. A path $u - v$ is called a strong path if it contains only strong edge [5]. If $\rho(u, v) > 0$, then u and v are called neighbors. Also v is called a strong neighbor if edge (u, v) is strong. The set of all neighbors of u is denoted by $N(u)$ and the set of all strong neighbors of u is denoted by $N_s(u)$. A vertex u is a fuzzy end vertex of G if it has exactly one strong neighbor in G .

A strong path P from u to v is a $u - v$ geodesic if there is no shorter strong path from u to v and the length of $u - v$ geodesic is the geodesic distance from u to v denoted by $d_s(u, v)$ [6]. Thus for all edge (u, v) which depending on the $CONN_G(u, v)$ of an edge (u, v) in a fuzzy graph G , strong edge are further classified as α -strong and β -strong and the remaining edges are termed as δ -edge [4] as follows.

An edge (u, v) in G is called α -strong if $\rho(u, v) > CONN_{G-uv}(u, v)$. An edge (uv) in G is called β -strong if $\rho(u, v) = CONN_{G-uv}(u, v)$, an edge (uv) in G is called δ -edge if $\rho(u, v) < CONN_{G-uv}(u, v)$ [7, 6, 8, 9, 4]. A δ -edge (u, v) is called a δ^* -edge if $\rho(u, v) > \rho(x, y)$ where (x, y) is a weakest edge of G . In a fuzzy graph G , $d_s : V \times V \rightarrow [0, 1]$ is a metric on V . i.e. $\forall u, v, w \in V$ (1), $d_s(u, v) \geq 0$; $\forall u, v \in V$; (2), $d_s(u, v) = 0$ if and only if $u = v$, (3), $d_s(u, v) = d_s(v, u)$ and (4), $d_s(u, v) \leq d_s(u, w) + d_s(w, v)$ [10]

Note that, the fuzzy subgraph H obtained by deleting the edge (u, v) from a fuzzy graph G . Let G be a graph with vertex set $V(G)$ and edge set $E(G)$, the distance $d_s(u, v)$ between two vertices $u, v \in V(G)$ is the minimum number of edges on a path in G between u and v . Let G be a connected fuzzy graph, for any path $P : u_0 - u_1 - u_2 - u_3 - \dots - u_n$, the length of P is denoted by $L(P)$, if $n = 0$, define $L(P) = 0$ and for $n \geq 1$, $L(P)$ is defined as the sum of the weights of the edges in P ,

$$L(P) = \sum_{i=1}^m \rho(u_i, u_{i+1}).$$

For any two vertices u and v in G , let $P = \{P_i : P_i \text{ is a } u - v \text{ path}, i = 1, 2, 3, \dots\}$. The sum distance between u and v is defined as $d_s(u, v) = \min\{L(P_i) : P_i \in P; i = 1, 2, \dots\}$.

Let G be a connected fuzzy graph, the Wiener index $W(G)$ of G is defined by

$$W(G) = \sum_{u, v \subseteq V(G)} d_s(u, v).$$

The average distance $\mu(G)$ between the vertices of G is defined by

$$\mu(G) = \frac{W(G)}{C_2^n}.$$

The Wiener index of fuzzy path P is defined by

$$W(P) = \sum_{i=1}^{n-1} i(n-i)\rho_i$$

see [11, 8, 12] and [13]. Let $G = (\mu, \rho)$ be a fuzzy graph on V , let $u, v \in V$ we say that u dominates v in G if there exists a strong edge between them. A set D of vertices of G is strong dominating set of G if every vertices of $V(G) - D$ is a strong neighbor of some vertex in D . The weight of a strong dominating set D is defined as $w(G) = \sum_{u \in D} \rho(u, v)$, where $\rho(u, v)$ is the minimum of the membership values (weights) of the strong edges incident on u . The strong domination number of a fuzzy graph G defined as the minimum weight of strong dominating sets of G and it is denoted by $\gamma_s(G)$ or simply γ_s .

3 Wiener Index of Fuzzy Graph by Using Strong Domination

Remark 3.1. Let G be fuzzy graph;

1. If G is not strong fuzzy cycle. Then $\mu(G) \geq \gamma_s$.
2. If G is a strong fuzzy cycle, Then $\mu(G) \leq \gamma_s$

Theorem 3.2. for any fuzzy path

$$W(P) \leq 2mq - (m + 1)\gamma_s(P)$$

Where $m = |E^*|$.

Proof. Let G be a fuzzy path with length L . Then the sum strong distance between u and w defined as $d_s(u, w) = \min\{L(P_i); P_i \in P, i = 1, 2, \dots\}$. So $d_s(u, w) \leq L(P_i) = \sum_{i=1}^m \rho(u_i, u_{i+1}) \leq q$.
 Taken sum

$$\sum_{i=1}^m d_s(u, w) \leq mq \dots \dots \dots (1).$$

Let D be strong dominating set of P . Then $\gamma_s(P) \leq \frac{q}{2}$

$$\implies (m + 2) \leq \frac{(m + 2)q}{2} \leq mq \dots \dots \dots (2).$$

From (1) and (2), we get

$$\sum_{i=1}^m d_s(u, w) \leq 2mq - (m + 2)\gamma_s \dots \dots \dots (3).$$

By triangle enquality $d_s(u, v) \leq d_s(u, w) + d_s(w, v)$ we have

$$\sum_{i=1}^m d_s(u, u) \leq \sum_{i=1}^m d_s(u, w) + \sum_{i=1}^m d_s(w, v) \dots \dots \dots (4).$$

Now we assume wv be a strong edge with minimum wight adjacent to at all vertices in strong dominating set of G . Then $d_s(w, v) = \rho(w, v)$.

Therefor,

$$\sum_{i=1}^m d_s(w, v) = \gamma_s \dots \dots \dots (5).$$

Substitute from (3) and (5) in (4), we obtain

$$W(P) \leq 2mq - (m + 1)\gamma_s$$

□

Example 3.3. consider the $G = (V, E)$ be path fuzzy graph, with $\mu(v) = 0.5$ for all $v \in V(G)$ gevin in the Fig. 1.

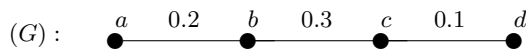


Fig. 1.

In figure 1, the vertex subsets $D_{s1} = \{b, d\}$, $D_{s2} = \{a, d\}$, $D_{s3} = \{a, c\}$, $D_{s4} = \{b, c\}$ are strong dominating set of G . Hence $\gamma_s = \min\{\gamma_{s1}, \gamma_{s2}, \gamma_{s3}\}$ so $\gamma_s(G) = 0.3$ and $m = |E^*| = 3$. Then the Wiener index of G

$$W(P) \leq 2mq - (m + 1)\gamma_s = 2 * 3 * 0.8 - (3 + 1) * 0.3 = 3.6$$

Also, the average of a path P is

$$\mu(P) \leq \frac{W(G)}{C_2^n} = \frac{2W(G)}{n(n-1)} = \frac{2 * 3.6}{4 * 3} = 0.6$$

Theorem 3.4. For any strong cycle fuzzy graph,

$$W(C_n) \leq \frac{m(mq - \gamma_s)}{8} + \frac{\gamma_s}{2}; \quad m = |E^*|.$$

Proof. Let G be a strong cycle fuzzy graph, the maximum length of geodesic in $C_n = \frac{m}{2}$.

So $d_s(u, w) = \frac{mq_1}{4}$, for $u, w \in P_{\frac{m}{2}}$.

Therefore, $q_1 = \frac{q}{2}$

Taken the sum, we have

$$\sum_{i=1}^m d_s(u, w) \leq \frac{m^2 q}{16} \dots\dots\dots(1).$$

Since $\gamma_s \leq \frac{q}{2}$, then

$$\frac{m}{8} \gamma_s \leq \frac{mq}{16} \leq \frac{m^2 q}{16} \dots\dots\dots(2).$$

From (1) and (2), we obtain,

$$\sum_{(u,w) \in P_{\frac{m}{2}}} d_s(u, w) \leq \frac{m(mq - \gamma_s)}{8} \dots\dots\dots(3).$$

Since $d_s(u, v) \leq d_s(u, w) + d_s(w, v)$, we have

$$\sum_{i=1}^m d_s(u, v) \leq \sum_{i=1}^m d_s(u, w) + \sum_{i=1}^m d_s(w, v)$$

We assume wv be strong edge with minimum wight adjacent to at all vertices in dominating set D_s of $P_{\frac{m}{2}}$, Then $d_s(w, v) = \mu(w, v)$ which impleis

$$\sum_{i=1}^m d_s(w, v) = \frac{\gamma_s}{2} \dots\dots\dots(5).$$

Substitute from (5) and (3) in (4), we obtain

$$W(C_n) \leq \frac{m(mq - \gamma_s)}{8} + \frac{\gamma_s}{2}$$

□

Example 3.5. Consider the strong cycle fuzzy graph $G = (V, E)$, with $\mu(v) = 1$ for all $v \in V(G)$ and all edges are α -strong gevin in the Fig. 2.

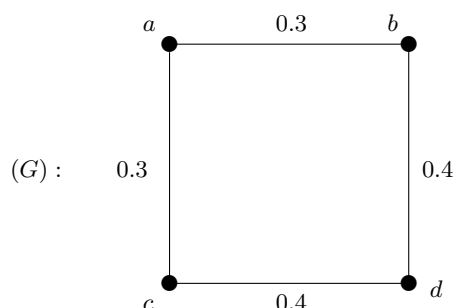


Fig. 2.

In Fig. 2. we have the strong dominating set $D = \{a, b\}$, $q = 1.4$, $\gamma_s(G) = 0.6$ and $m = |E^*| = 4$. Then the Wiener index of G

$$W(C_n) \leq \frac{m(mq - \gamma_s)}{8} + \frac{\gamma_s}{2} = \frac{4(4 * 1.4 - 0.6)}{8} + \frac{0.6}{2} = 2.8.$$

Also, the average of a strong fuzzy cycle C is

$$\mu(C) \leq \frac{W(C)}{C_2^n} = \frac{2W(C)}{n(n-1)} = \frac{2 * 2.8}{4 * 3} = 0.5$$

Theorem 3.6. For any fuzzy graph G has not strong cycle fuzzy graph,

$$W(G) \leq 2m_s q_s - (m_s + 1)\gamma_s.$$

Where m_s is the number of strong edge in G , and q_s is the sum of strong edge

Proof. Let G be a fuzzy graph, with n vertices and m edges has not strong cycle and has at least one δ -edge. Then G has a path joining v_1 and v_n we obtained it after delet δ -edge. So the path in G is strong path has not δ -edge and has m_s strong edge. Therefore, by Theorem (3.2),

$$W(G) \leq 2m_s q_s - (m_s + 1)\gamma_s.$$

□

Example 3.7. Consider the fuzzy graph $G = (V, E)$, with $\mu(v) = 1$ for all $v \in V(G)$ and all edges are α -strong except (cd) is δ -edge given in the Fig. 3.

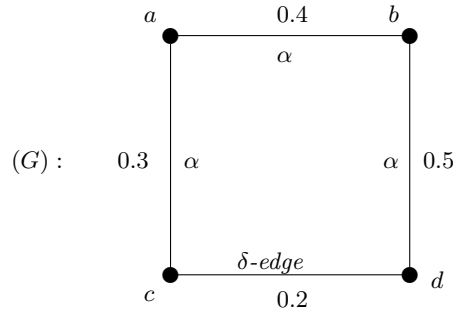


Fig. 3.

In Fig. 3, we have, $q = 1.4$, $q_s = 1.2$, $\gamma_s(G) = 0.7$ and $m = |E^*| = 4$, $m_s = 3$. Then the Wiener index of G

$$W(G) \leq 2m_s q_s - (m_s + 1)\gamma_s = 2 * 3 * 1.2 - (3 + 1) * 0.7 = 4.4$$

Also, the average of G is

$$\mu(G) \leq \frac{W(G)}{C_2^n} = \frac{2W(G)}{n(n-1)} = \frac{2 * 4.4}{4 * 3} = 0.73.$$

Theorem 3.8. For any fuzzy graph G without δ -edge and contain k strong cycle fuzzy graph, such that $k \geq 2$.

$$W(G) \leq \frac{m(mq - \gamma_s)}{8K}.$$

Where $m = |E^*|$

Proof. Let G be a fuzzy graph without δ -edge and contain $K \geq 2$ strong cycle. Then $G = \cup_{i=2}^k C_i$. By Theorem (3.4),

$$W(G) \leq \frac{m(mq - \gamma_s)}{8}.$$

We have, $m = \sum_{i=2}^k \mu_i$. Hence

$$W(G) \leq \frac{m(mq - \gamma_s)}{8K} + \frac{\gamma_s}{2K}.$$

Since $\frac{\gamma_s}{2K}$ is very small. Then

$$W(G) \leq \frac{m(mq - \gamma_s)}{8K}.$$

□

Example 3.9. Consider the fuzzy graph G with $\mu(v) = 1$ for all $v \in V(G)$ given in the Fig. 4.

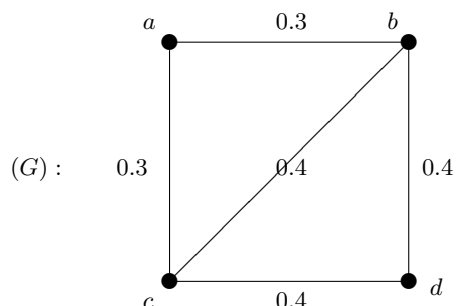


Fig. 4.

From Fig. 4. we have, $q = 1.8, \gamma_s(G) = 0.3$ and $m = |E^*| = 5, K = 2$. Then

$$W(G) \leq \frac{m(mq - \gamma_s)}{8K} = \frac{5(5 * 1.8 - 0.3)}{8 * 2} = 2.72$$

Also, the average of a fuzzy graph G is

$$\mu(G) \leq \frac{W(G)}{C_2^n} = \frac{2W(G)}{n(n-1)} = \frac{2 * 2.72}{4 * 3} = 0.45$$

Theorem 3.10. Let G be a fuzzy graph without δ -edge, contains strong cycle fuzzy graph and strong fuzzy path. Then

$$W(G) \leq \frac{m(mq - \gamma_s)}{8} + \frac{q}{2}.$$

Proof. By Theorem (3.4) and Theorem (3.2), we have

$$W(G) \leq \frac{m(mq - \gamma_s)}{8} + \frac{q}{2} + (2mq - (m+1)\gamma_s).$$

There exists a vertex u which is a common strong cycle and strong path. Let u belong to a strong cycle. Therefore the path contain $(V(P) - \{u\})$, also the path P contains m edge such $m = m_c + m_P, q = q_C + q_P$.

Then $2m_P q_P - (m_P + 1)\gamma_s(P) \leq \frac{q}{2}$.

Hence

$$W(G) \leq \frac{m(mq - \gamma_s)}{8} + \frac{q}{2}.$$

□

Example 3.11. Consider the fuzzy graph G with $\mu(v) = 1$ for all $v \in V(G)$ given in the Fig. 5.

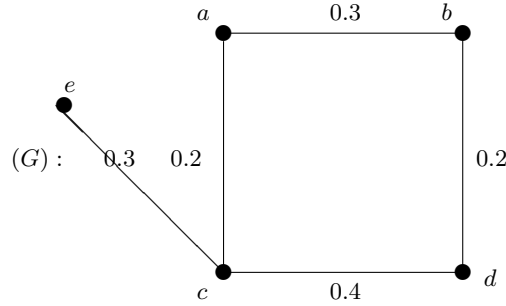


Fig. 5.

From Fig. 5, we have, $q = 1.4$, $\gamma_s(G) = 0.4$ and $m = |E^*| = 5$. Then

$$W(G) \leq \frac{m(mq - \gamma_s)}{8} = \frac{5(5 * 1.4 - 0.4)}{8} + \frac{1.4}{2} = 4.8$$

Also, the average of a strong fuzzy cycle C is

$$\mu(G) \leq \frac{W(G)}{C_2^n} = \frac{2W(G)}{n(n-1)} = \frac{2 * 4.8}{5 * 4} = 0.48$$

Theorem 3.12. For any fuzzy graph G , with at least one δ -edge and has strong fuzzy cycle. Then

$$W(G) \leq \frac{m_s(m_s q_s - \gamma_s)}{8}.$$

Where m_s is the number of strong edge of G .

Proof. Let G be a fuzzy graph with δ -edge and G has strong cycle. Then G contains strong fuzzy cycle and fuzzy subgraph obtained by deleting δ -edge. So by Theorems (3.4), (3.2) and Theorem (3.10), we have

$$W(G) \leq \frac{m_c(m_c q_c - \gamma_s)}{8} + (2m_H q - (m_H + 1)\gamma_s).$$

Hence

$$W(G) \leq \frac{m_s(m_s q_s - \gamma_s)}{8}$$

□

Example 3.13. Consider the fuzzy graph G , with $\mu(v) = 1$ for all $v \in V(G)$ given in Fig.6.

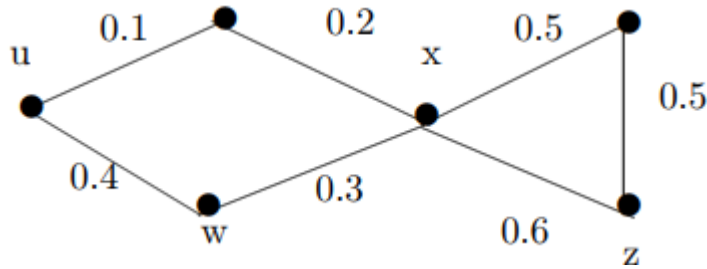


Fig. 6.

From Fig. 6, we have, $q = 2.6$, $q_s = 2.5$, $\gamma_s(G) = 0.5$ and $m = |E^*| = 7$, $m_s = 6$. Then

$$W(G) \leq \frac{6(6 * 2.5 - 0.5)}{8} = 10.9.$$

Also, the average of a fuzzy graph G is,

$$\mu(G) \leq \frac{W(G)}{C_2^n} = \frac{2W(G)}{n(n-1)} = \frac{2 * 10.9}{6 * 5} = 0.73.$$

4 Application

In this section, we will present a parctical example about the above-mentioned results and how they save time and efforts in calculating the Wiener index. The first value will be considered as the memmbership edge that we will use in our solutions.



Fig. 7.

In Fig. 7, The membership edges of the path from China \rightarrow Columbia \rightarrow Guatemala \rightarrow Mexico \rightarrow United States route is $\{0.19, 0.3, 0.32, 0.47\}$, and we have the strong dominating set $D = \{colombia, Mixico\}$ and $m = 4$, $q = 1.28$, $\gamma_s = 0.51$. Therefore, the Wiener index $W(G) \leq 7.69$ and average $\mu(G) \leq 0.77$. Similar calculations in India \rightarrow Guatemala \rightarrow Mexico \rightarrow United States route, we can see that $D = \{Guatemala, Mixico\}$, $m = 3$, $q = 1.05$, $\gamma_s = 0.58$. Hence $W(G) \leq 3.98$, $\mu(G) \leq 0.66$. Along Ethiopia \rightarrow S. Africa \rightarrow Brazil \rightarrow Ecuador \rightarrow Mexico \rightarrow United States route, $D = \{S.Africa, Mixico\}$, $m = 5$, $q = 1.63$, $\gamma_s = 0.5$. So, $W(G) \leq 13.3$, $\mu(G) \leq 0.89$. Along Somalia \rightarrow EAU \rightarrow Russia \rightarrow Cuba \rightarrow Columbia \rightarrow Mexico \rightarrow United States route, $D = \{Somalia, Cuba, Mixico\}$, $m = 6$, $q = 1.53$, $\gamma_s = 0.69$. Hence $W(G) \leq 13.53$, $\mu(G) \leq 0.64$. Along Nigeria \rightarrow Spain \rightarrow Cuba \rightarrow Columbia \rightarrow Mexico \rightarrow United States route, $D = \{Spain, Mixico\}$, $m = 5$, $q = 1.6$, $\gamma_s = 0.62$. Then $W(G) \leq 12.28$, $\mu(G) \leq 0.82$. The method in [14] and the method in this paper all agree that Somalia is the source country of the path to the United States of strongest susceptibility.

5 Conclusions

This paper has focused on calculating the Wiener index of a fuzzy graph G by using the strong domination number γ_s , if the number of edges of the fuzzy graph are very large. It's an easy way to save time and efforts by reducing a lot of procedure to calculate the Wiener index. So we use a new method that is strong domination to find and calculate the Wiener index. The Wiener index for some standard fuzzy graphs such as a fuzzy path and fuzzy cycle are given. The relationship of $\mu(G)$ and γ_s was investigated.

Competing Interests

Authors have declared that no competing interests exist.

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