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Analysis of Means by Ranks: Modification and Application in Contract Acquisition Analysis

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

Analysis of Means by Ranks is a nonparametric statistical test procedure that was developed in Bakir (1989) but has rarely been applied in practice. This paper modifies and applies Analysis of Means by Ranks to a case study data involving the comparison of three contract proposals. For comparison purposes, we analyze the same data using the well-known Analysis of Variance, Analysis of Means, and the Kruskal-Wallis test. Analysis of Variance and Analysis of Means are two parametric (assume data to be samples from normal populations) test procedures whereas Kruskal-Wallis and Analysis of Means by Ranks are two nonparametric (or distribution-free) procedures. This paper shows that the parametric tests fail to detect a significant difference among three contract proposals, while the nonparametric tests do. The conclusions of the parametric tests are in doubt because a descriptive statistics analysis indicates that the required normality assumption is in doubt; the nonparametric conclusions are more trustful because the normality assumption is not required by nonparametric procedures.

Keywords: *Analysis of means; analysis of means by ranks; analysis of variance; kruskal-wallis test; nonparametric.*

1 Introduction

This paper modifies Analysis of Means by Ranks (ANOMR) and applies it to a case study in contract acquisition analysis. ANOMR is a nonparametric statistical test of hypotheses procedure that was developed

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in Bakir [1] but has rarely been used in practice. The modified ANOMR is then applied to a case study data that involves the comparison of contract proposals. The paper also analyzes the same data using some well-known parametric tests (assume data are samples from normal probability distributions) and nonparametric tests that do not require the normality assumption. The parametric tests are Analysis of Variance (ANOVA) and Analysis of Means (ANOM); the nonparametric tests are Analysis of Means by Ranks (ANOMR) and the Kruskal-Wallis (K-W). The paper uses data that appear in a case study of a course (CON 270) offered by the Defense Acquisition University (DAU). The case study stipulates that a Request for Proposal (RFP) was posted to perform inspection and calibration of some instruments. The Government has asked for historical performance data on last 30 maintenance operations of three contractors A, B, and C who submitted their data as shown in Table 1. The contractors based their proposals on the following "typical" hours per maintenance operation: 123 for A, 123 for B, and 118 for C. Descriptive statistics in Fig. 2 shows that contractors A and B reported the mean (=median) of their sample data while C reported the smallest of two modes of his sample data, Obviously, it is misleading to make evaluations based on the contractors' reported "typical" hours only. A substantial part of contracts acquisition analysis in CON 270 uses descriptive statistics (central tendency, variation, shape, histograms, etc.) to evaluate and compare contract proposals. Descriptive statistics alone, however, does not provide clear-cut decision rules to detect significant differences among contractors. This paper appends the descriptive analysis by the above-mentioned significance test procedures.

Table 1. Maintenance Hours for Contractors A, B, and C

#	A	B	C
1	122.00	105.00	132.00
2	124.00	124.00	134.00
3	118.00	116.00	118.00
4	123.00	123.00	131.00
5	123.00	123.00	133.50
6	125.00	123.00	125.00
7	119.00	102.00	132.00
8	118.00	107.00	133.00
9	122.00	140.00	122.00
10	126.00	137.00	135.00
11	129.00	131.00	97.00
12	128.00	128.00	128.00
13	118.00	141.00	136.00
14	127.00	115.00	127.00
15	130.00	130.00	130.00
16	123.00	116.00	135.00
17	120.00	145.00	120.00
18	120.00	120.00	106.00
19	119.00	119.00	130.00
20	126.00	100.00	126.00
21	125.00	106.00	135.00
22	124.00	113.00	133.00
23	122.00	144.00	134.00
24	121.00	103.00	118.00
25	115.00	142.00	118.00
26	121.00	111.00	134.50
27	125.00	133.00	125.00
28	123.00	130.00	136.00
29	124.00	135.00	136.50
30	130.00	128.00	128.00

Source: Defense Acquisition University: https://icatalog.dau.edu/onlinecatalog/courses.aspx?crs_id=1838

Now we formulate the general problem of comparing several populations. Consider J independent random samples $\left(y_{1j},\ y_{2j},\ \cdots, y_{n_j,j}\right), j = 1, 2, \cdots, J$ that have been drawn from J continuous populations $F_1(y), F_2(y), \ldots, F_J(y)$. The data structure of the samples is shown in Table 2.

Table 2. Data structure for a single factor comparison

	Samples (Groups or Factor Levels)					
$i \setminus j$	1	2	…	j	…	J
1	y_{11}	y_{12}	…	y_{ij}	…	y_{1J}
2	y_{21}	y_{22}	…	y_{2j}	…	y_{2J}
…	…	…	…	…	…	…
i	y_{i1}	y_{i2}	…	y_{ij}	…	y_{iJ}
…	…	…	…	…	…	…
Sample Total	$y_{.1}$	$y_{.2}$	…	$y_{.j}$		$y_{.J}$
Sample Mean	$\bar{y}_{.1}$	$\bar{y}_{.2}$	…	$\bar{y}_{.j}$	…	$\bar{y}_{.J}$

The j^th sample total and mean are, respectively

$$y_{.j} = \sum_{i=1}^{n_j} y_{ij} \quad \text{and} \quad \bar{y}_{.j} = \frac{1}{n_j}\sum_{i=1}^{n_j} y_{ij}, \quad j = 1, 2, \ldots, J. \tag{1}$$

If $N = \sum_{j=1}^{J} n_j$ represents the total number of observations in the combined sample, then the grand total and grand mean of the combined sample are

$$y_{..} = \sum_{j=1}^{J}\sum_{i=1}^{n_j} y_{ij} \quad \text{and} \quad \bar{y}_{..} = \frac{1}{N} y_{..} \tag{2}$$

The general purpose of most comparison test procedures is to detect statistically significant differences among the J populations. Some procedures are specifically designed to detect significant differences in the locations (central tendency measures), some for detecting differences in the spread (or variation), and others for detecting broad-type of differences in the populations. Most statistical test procedures impose some restrictive assumptions on the probability distributions of the populations. Parametric tests assume the populations have normal distributions; nonparametric tests, do not assume any type of probability distribution on the data.

The well-known ANOVA and the less-well-known ANOM are two parametric tests designed to detect significant differences in the central values (means) of the populations. The K-W and ANOMR are two nonparametric tests designed to detect significant differences in the central locations of the populations. To detect broad-type of differences (not necessarily center or variance) among several populations, one can use extensions of the two-sample Kolmogorov-Smirnov test or the Cramer-von Mises test; for a reference see Conover [2] A thorough textbook on nonparametric statistics is Gibbons and Chakraborti [3].

Results in this paper show that the parametric tests (ANOVA and ANOM) fail to detect significant difference among the three contractors' data in Table 1, while the nonparametric do. The parametric conclusion is in doubt because the descriptive statistics analysis in Section 2 shows that contractors B and C data are far from being samples from normal populations; thus, the parametric assumption of normality is not satisfied. The nonparametric procedures (ANOMR and K-W) do detect significant difference among the contractors; this conclusion is more trustful because the data does not have to be normally distributed for nonparametric procedures. Several authors have discussed the implications of parametric and nonparametric test statistics in data analysis. Murray [4] studied the effect of using parametric or nonparametric tests on data of the Likert scale. Kim [5] discussed the benefits of using nonparametric methods in clinical trials. Egboro [6] demonstrated that incorrect choice between parametric and nonparametric tests may lead to incorrect conclusions.

Section 2 of this paper presents the descriptive statistics analysis of the data in Table 1. Sections 3 and 4 present the parametric test procedures: ANOVA and ANOM. Sections 5 and 6 present the nonparametric tests: ANOMR and K-S. Section 7 contains summary and conclusion.

2 Descriptive Statistics Analysis

Descriptive (or summary) statistics is a collection of numerical measures and graphics that can describe important characteristics of a numerical data set. Table 3 lists some of the common descriptive statistics measures including measures of central tendency, measures of variation, and measures of shape. Graphical descriptive statistics includes drawing trend lines, histograms, and box plots. The summary measures of a data set are often compared to those of the normal probability distribution. The normal probability density function (pdf) and its curve (bell-curve) are given in Formula 3 and Figure 1, respectively.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}; \quad -\infty < x < \infty\,,\ -\infty < \mu < \infty \text{, and } \sigma > 0\ . \qquad (3)$$

Mean $E(X) = \mu\ \ and\ variance = \sigma^2$.

The graph of the normal pdf, in Fig. 1, is symmetric (about the mean) and bell-shaped with tails that become asymptotic to the horizontal axis.

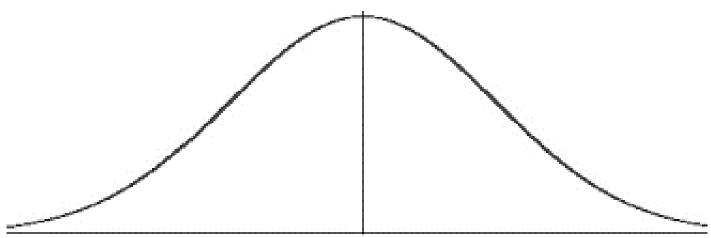


Figure 1. Normal Distribution (Bell) Curve

For a normal population, the mean (or expected value), the median, and the mode have identical value; the skewness and excess kurtosis have zero values. A data set with negative skewness indicates an asymmetric curve with a long tail extending to the negative side (left); a positive skewness indicates an asymmetric curve with a long tail extending to the positive side (right). Data with zero excess kurtosis (such as the normal) is termed mesokurtic, with negative excess kurtosis is termed platykurtic (flatter peak than the normal), and with positive excess kurtosis is termed leptokurtic (sharper peak than the normal.)

The descriptive statistics in Fig. 2 show that data sets A and B have same mean of 123.0, and set C has a mean of 127.6. For data set A, the histogram, skewness = 0.1 and kurtosis = -0.4) indicate that it is close to being normally distributed. Data set B is symmetric (skewness = - 0.1), but its histogram and excess kurtosis of -1.0 indicate a flatter curve (platykurtic) than the normal. Data set C with skewness = -1.8 and excess kurtosis of 3.5 indicate an asymmetric peaked curve (leptokurtic). In conclusion, although the data sets have approximately similar means, they greatly differ in the shape of their probability distributions. Further, the data sets differ in their variability with coefficients of variations 3.1% for A, 11% for B, and 7.2 for C.

Based on the descriptive statistics analysis in Fig. 2, it is quite rational to make a preliminary conclusion that the historical performance data of the three contractors differ in more than one aspect. In the ensuing sections of this paper, we use more sophisticated statistical inference test procedures (ANOVA, ANOM, ANOMR, and K-W) to reach more concrete conclusions.

Table 3. Common Descriptive Summary Statistics Measures				
Measures of	**NAME: sample ...**	**Symbol**	**Formula**	**EXCEL 2013 Function**
I. Central Tendency	mean	$\bar{y}$	$\bar{y} = \frac{\sum y}{n}$	AVERAGE( )
Indicate a central (average, middle, typical) value	median	$\tilde{y}$	middle value of the ordered data	MEDIAN( )
	mode	$\grave{y}$	most frequent value in the data	MODE.MULT( ), MODE.SNGL( )
	midrange		(max + min)/2	(MAX()+MIN())/2
II. POSITION	standardized or z score	z	$z = \left(\frac{y - \bar{y}}{s}\right)$	STANDARDIZE(x,mean,standard_dev)
Indicate the relative position of a value	percentiles	P	a value below which a certain percentage of the data fall	PERCENTILE.INC( )
	quartiles (special percentiles)	Q1, Q2, Q3	below Q1, two quarters (50%) below Q2 and 3 quarters (75%) below Q3	QUARTILE.INC( )
III. VARIATION, VARIABILITY or DISPERSION	range	range	Type equation here. max-min	MAX( ) -MIN( )
Indicate the spread (variation)	variance	s^2	$s^2 = \frac{\sum(y - \bar{y})^2}{n - 1}$	VAR.S( )
	standard deviation	s	$s = \sqrt{variance}$	STDEV.S( )
	Standard error	SE	$standard\ deviation/\sqrt{n}$	
	coefficient of variation	cv	$cv = \left(\frac{s}{\bar{y}}\right) * 100\ \%$	(STDEV()/AVERAGE()) *100
	mean absolute deviation	mad	$mad = \frac{\sum\lvert y - \bar{y}\rvert}{n}$	AVEDEV( )
	interquartile range		Q3 - Q1	
IV. SHAPE	skewness	skew	$\frac{n\sum z^3}{(n-1)(n-2)}$	SKEW( )
Indicate the shape of a data	kurtosis		$\frac{\sum z^4}{n}$	Not in Excel 2013
	excess kurtosis	Kurt	$\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum z^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$	KURT( )

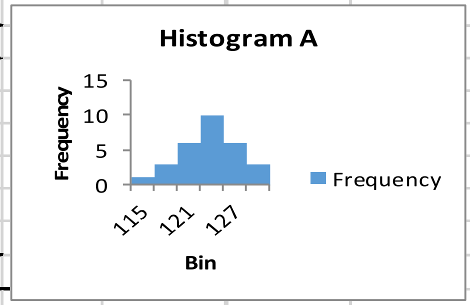
Figure 2. Descriptive Statistics of Historical Performance Data of Contractors A, B, and C

Historical Data Contractor, A, B, and C

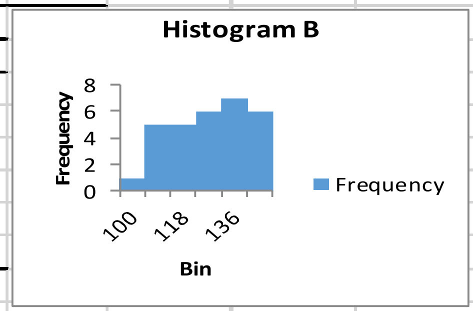
A	B	C
122.00	105.00	132.00
124.00	124.00	134.00
118.00	116.00	118.00
123.00	123.00	131.00
123.00	123.00	133.50
125.00	123.00	125.00
119.00	102.00	132.00
118.00	107.00	133.00
122.00	140.00	122.00
126.00	137.00	135.00
129.00	131.00	97.00
128.00	128.00	128.00
118.00	141.00	136.00
127.00	115.00	127.00
130.00	130.00	130.00
123.00	116.00	135.00
120.00	145.00	120.00
120.00	120.00	106.00
119.00	119.00	130.00
126.00	100.00	126.00
125.00	106.00	135.00
124.00	113.00	133.00
122.00	144.00	134.00
121.00	103.00	118.00
115.00	142.00	118.00
121.00	111.00	134.50
125.00	133.00	125.00
123.00	130.00	136.00
124.00	135.00	136.50
130.00	128.00	128.00

Note: Bins are the upper class limis of a histogram

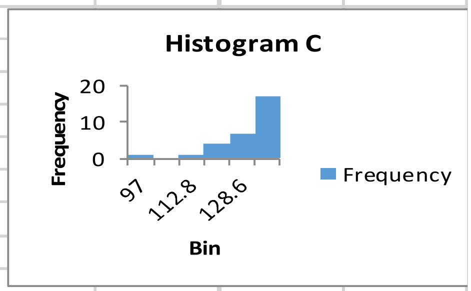
Bin A	*Frequency*
115	1
118	3
121	6
124	10
127	6
More	3



Bin B	*Frequency*
100	1
109	5
118	5
127	6
136	7
More	6



Bin C	*Frequency*
97	1
104.9	0
112.8	1
120.7	4
128.6	7
More	17



Descriptive Statiatics for Contractors A, B, and C

	A	*B*	*C*
Mean	123.0	123.0	127.6
Standard Error	0.7	2.4	1.7
Median	123.0	123.0	130.5
Mode	123.0	123.0	118.0
Standard Deviation	3.8	13.4	9.2
Sample Variance	14.1	180.1	84.6
Kurtosis	-0.4	-1.0	3.5
Skewness	0.1	-0.1	-1.8
Range	15.0	45.0	39.5
Minimum	115.0	100.0	97.0
Maximum	130.0	145.0	136.5
Sum	3690.0	3690.0	3828.5
Count	30	30	30
Coeff of Variation	3.05%	10.9%	7.2%

3 Analysis of Variance (ANOVA)

The single-factor (or one-way) ANOVA procedure assumes that the data represent independent random samples drawn from normal populations having the same variance. If Δ_j represents the central value (mean or median) of the jth population, then ANOVA is designed to test the following null (H_0) and alternative (H_a) statistical hypotheses:

$$H_0: \Delta_1 = \Delta_2 = \cdots = \Delta_j = \cdots = \Delta_J \quad \text{and} \quad H_a: not\ all\ \Delta's\ are\ equal \tag{4}$$

To test for a significant difference among the population means, ANOVA decomposes the total variation in the data into variation between groups and variation within groups (sometimes called error or residual):

Total Variation = Between Groups Variation + Within Groups Variation

$$\sum_{j=1}^{J}\sum_{i=1}^{n_j}\left(y_{ij} - \bar{y}_{..}\right)^2 = \sum_{j=1}^{J} n_j\left(\bar{y}_{.j} - \bar{y}_{..}\right)^2 + \sum_{j=1}^{J}\sum_{i=1}^{n_j}\left(y_{ij} - \bar{y}_{.j}\right)^2 \tag{5}$$

The terms in Formula 5 are called sums of squares (*SS*): Total sum of squares (SS_{total}) on the left, Between Groups sum of squares ($SS_{between}$) in the middle, and Within Groups (or error) sum of squares (SS_{error}) on the extreme right. These sums of squares are then divided by their corresponding degrees of freedom (*df*) to produce mean squares (*MS*). The test statistic of the single-factor ANOVA procedure is $F = MS_{between}/MS_{error}$ which has an F-distribution with ($J-1$, $N-J$) degrees of freedom under the null hypothesis of no significance difference; see Table 4.

Table 4. Single factor ANOVA table

Source of Variation	***SS***	***df***	***MS***	***F***	***P*-value**
Between Groups	$SS_{between}$	$J - 1$	$SS_{between}/(J-1)$	$MS_{between}/MS_{error}$	Computer generated
Within Groups (error)	SS_{error}	$N - J$	$SS_{error}/(N-J)$		
Total	SS_{total}	$N - 1$			

The ANOVA test procedure is judged significant if the *P*-value of the *F*-test statistic is small (less than a commonly assumed significance level of 0.05). Table 5 shows Excel's Single-Factor ANOVA output when applied to the data of Table 1.

Table 5. Results of single-factor ANOVA for Data in table 1

Source of Variation	***SS***	***df***	***MS***	***F***	***P*-value**
Between Groups	426.27	2	213.14	2.294	0.107
Within Groups (error)	8084.34	87	92.92		
Total	8510.61	89			

The *P*-value of 0.107 in Table 5 results in concluding no significant difference among the three contractors; a conclusion that contradicts the findings of the descriptive statistics of Section 2. We should doubt the ANOVA conclusion because the descriptive analysis in Section 2 shows that data sets B and C are not normally distributed.

4 Analysis of Means (ANOM)

Ott [7] developed the ANOM procedure to detect statistically significant differences among several population means as expressed in Formula (4). ANOM assumes that data represent independent random samples from normal populations having the same variance. ANOM plots the sample means on a chart that has a lower decision line (LDL), a central line (CL), and an upper decision line (UDL). The ANOM procedure judges the populations whose sample means fall below LDL or above UDL to be significantly different from the rest that fall within the decision lines. If all sample means fall within the decision lines, then there is no significant difference among the populations. A concise summary of ANOM and its variants can be found in Wheeler [8].

In this paper, we present the ANOM mathematical details when the sample sizes are equal (the balanced case); details of the unbalanced case can be found in Nelson [9] and Nelson et al. [10]. The decision lines for the balanced case of ANOM are

$$LDL = \bar{y}_{..} - h(\alpha; J, \nu)\sqrt{MS_{error}}\sqrt{\frac{J-1}{N}} \quad (6)$$

$$CL = \bar{y}_{..} \quad (7)$$

$$UDL = \bar{y}_{..} + h(\alpha; J, \nu)\sqrt{MS_{error}}\sqrt{\frac{J-1}{N}} \quad (8)$$

Nelson [11,12] provides tables for the critical values $h(\alpha; J, \nu)$ that depend on a desired significance level α, the number of samples *J*, and the error degrees of freedom, ν .

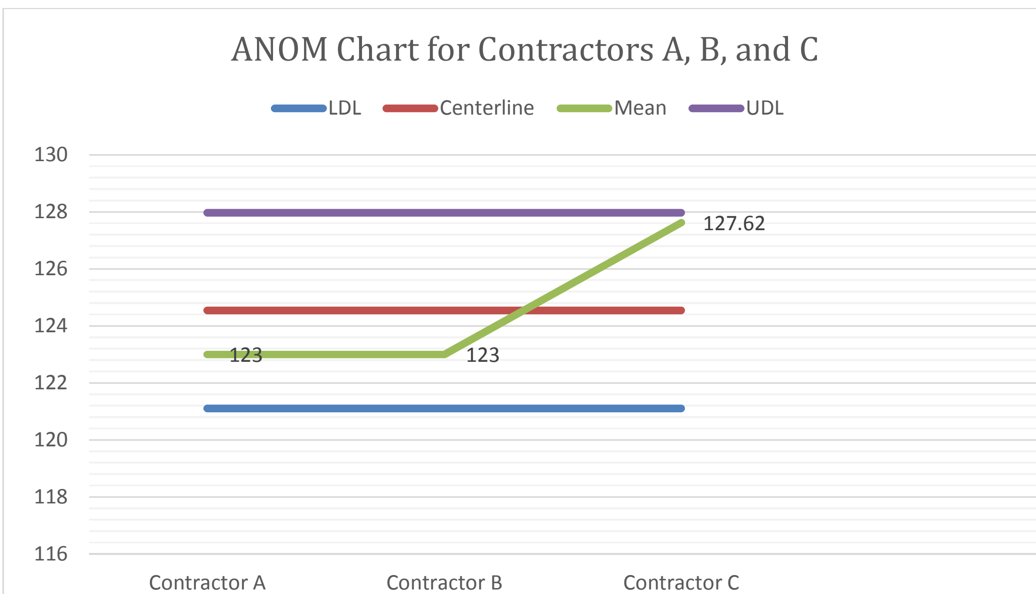


Fig. 3. ANOM chart for contractors A, B, and C

Now we apply ANOM to detect significant differences among the contractors' data in Table 1. From the ANOVA results in Table 5, we read, $MS_{error} = 92.92$, $\nu = 87$, and *J* = *3*. Also the means of contractors A, B, and C are: $\bar{y}_A = 123$, $\bar{y}_B = 123$, $\bar{y}_C = 127.62$, and the grand mean is $\bar{y}_{..} = 124.54$. Assuming significance level $\propto= 0.05$, tables in Nelson et al. [10] give the critical value $h(\alpha; J, \nu) = h(0.05; 3{,}87) = 2.39$, by interpolation. Therefore, the LDL, CL, and UDL of ANOM are:

$$LDL = 124.54 - 2.39\sqrt{92.92}\sqrt{\frac{3-1}{90}} = 121.106 .$$

$CL = 124.54$

$UDL = 124.54 + 2.39\sqrt{92.92}\sqrt{\frac{3-1}{90}} = 127.974$.

The ANOM chart in Fig. 3 shows that all sample means fall within the LDL and UDL; thus, no significant difference is detected by the ANOM procedure. However, this ANOM conclusion is doubtful or misleading because the assumption of normality required by ANOM is not satisfied as seen in the descriptive analysis of Section 2.

5 Analysis of Means by Ranks (ANOMR)

Bakir [1] developed ANOMR as a nonparametric analogue of the parametric ANOM. ANOMR is designed to test the hypotheses in Formula (4); it assumes that data represent independent random samples drawn from continuous (not necessarily normal) populations having the same variance. ANOMR replaces each observation y_{ij} by its rank, denoted by R_{ij}, in the combined sample of all observations. If some observations have the same value (tied), they are assigned the average of their ranks. Symbolically,

$$R_{ij} = \sum_{t=1}^{J}\sum_{k=1}^{n_t} U(y_{ij} - y_{kt}), \tag{9}$$

The indicator function $U(u) = 0$, or 1 when $u < 0$ or $u \geq 0$, respectively.

Calculate, $\bar{R}_{.j}$, the mean of the jth sample ranks by

$$\bar{R}_{.j} = \frac{1}{n_j}\sum_{i=1}^{n_j} R_{ij}\ , \qquad j = 1,2,\dots,J\ . \tag{10}$$

Then the grand mean, $\bar{R}_{..}$, of all ranks in the combined sample is

$$\bar{R}_{..} = \frac{1}{N}\sum_{j=1}^{J}\sum_{i=1}^{n_j} R_{ij} = \frac{(N+1)}{2}\ . \tag{11}$$

At a significance level, α, the decision rule of ANOMR is to reject the null hypothesis H_0 in Formula (4), if for any $j = 1,2,\cdots,J$

$$\left|\bar{R}_{.j} - \bar{R}_{..}\right| > c(\propto, J, n_1, n_2, \cdots, n_J), \tag{12}$$

where the critical values $c(\propto, J, n_1, n_2, \cdots, n_J)$ are determined to satisfy Formula (16), given below.

Graphical implementation of the decision rule in Formula (12) amounts to plotting the $\bar{R}_{.j}$'s on a chart that has the following decision lines:

$$LDL = \bar{R}_{..} - c(\alpha; J, n_1, n_2, \dots, n_J)\ , \tag{13}$$

$$CL = \bar{R}_{..}\ , \tag{14}$$

$$UDL = \bar{R}_{..} + c(\alpha; J, n_1, n_2, \dots, n_J)\ . \tag{15}$$

ANOMR decision rule becomes: Reject the null hypothesis H_0 and consider the populations with rank means $\bar{R}_{.j}$ falling outside the decision lines to be significantly different from the rest whose rank means fall within the decision lines. If all rank means fall within the decision lines, then there is no significant difference among the populations. The critical values $c(\alpha; J, n_1, n_2, \ldots, n_J)$ are determined to satisfy the condition that under H_0 ,

$$\Pr\left[\max_{1\le j\le J}\left\lfloor \bar{R}_{.j} - \bar{R}_{..}\right\rfloor \ge c(\alpha; J, n_1, n_2, \ldots, n_J)\right] = \propto \quad . \tag{16}$$

Bakir [1] Table 2 computed the exact critical values, $c(\alpha; J, n_1, n_2, \ldots, n_J)$ for comparing 3 or 4 samples each of size 4; however, this is not suitable for our data in Table 1 that consists of three samples of size 30 each.

5.1 Modification of ANOMR

We now develop a large sample (asymptotic) version of ANOMR that can be very useful at our current age of Big Data where sample sizes are extremely large.

For $j = 1, 2, \cdots, J$, define the standardized ranks $W_{.j} = (\bar{R}_{.j} - \bar{R}_{..})/\sqrt{\sigma_{jj}}$, where $\sigma_{jj} = (N - n_j)(N + 1)/12n_j$ is the variance of the j^th^ rank mean $\bar{R}_{.j}$. Based on results in Kruskal [13], Bakir [1] showed that the standardized ranks, $W_{.j}^{'}s$, have a joint asymptotic (as $N \to \infty, \frac{n_j}{N} \to a_{j>0}$) singular multivariate normal distribution. Assuming equal sample sizes (if not, use the average size (say, *n*) as the common sample size), the standardized rank means become

$$W_{.j} = (\bar{R}_{.j} - \bar{R}_{..})/\sqrt{\frac{(N-n)(N+1)}{12n}}, \tag{17}$$

At significance level, α, the decision rule of the asymptotic ANOMR is to reject the null hypothesisH_0, if for any $j = 1,2, \cdots, J$

$$|W_j| > \omega(\propto; J) . \tag{18}$$

The critical values $\omega(\propto; J)$ such that $Pr\left[\max_{1\le j\le J}\left\lfloor W_{.j}\right\rfloor \ge \omega(\propto; J)\right] = \propto$, represent upper percentage points of a singular equicorrelated multivariate normal distribution; they are available in Nelson [11]. Bakir [1] Table 4 reprints these critical values for $\propto = 0.1, 0.05, 0.01, 0.001$ and $J = 3, 4, \cdots, 20$.

The modified asymptotic lower and upper ANOMR decision lines become

$$LDL_{asymptotic} = \bar{R}_{..} - \omega(\propto; J)\sqrt{\frac{(N-n)(N+1)}{12n}} \quad , \tag{19}$$

$$CL = \bar{R}_{..} \quad , \tag{20}$$

$$UDL_{asymptotic} = \bar{R}_{..} + \omega(\propto; J)\sqrt{\frac{(N-n)(N+1)}{12n}} \quad . \tag{21}$$

5.2 Application of the Modified ANOMR

The following steps show the application of the modified ANOMR to the data in Table 1.

Step 1. Generate the ranks, R_{ij} in Formula (9): Treating Table 1 as one combined sample, assign rank 1 to the smallest data value and rank 30 to the largest data value. If some data have the same value (tied), they are assigned the average of their ranks. Also calculate the rank means $\bar{R}_{.j}$ and grand rank mean $\bar{R}_{..}$ as given in Table 6: $\bar{R}_{..} = 45.5$, $\bar{R}_{.1} = 37.55, \bar{R}_{.2} = 42.4, \bar{R}_{.3} = 56.55$.

Step 2. Calculate the decision lines according to Formulae (19), (20), and (21): For significance level $\propto = 5\%\ and\ J = 3, we\ read\ \omega(\propto; J) = 2.34$ in Bakir [1], Table 4.

Using the grand rank mean $\bar{R}_{..} = 45.5$, we calculate the following decision lines for the modified asymptotic ANOMR procedure:

$$LDL_{asymptotic} = \bar{R}_{..} - \omega(\propto; J)\sqrt{\frac{(N-n)(N+1)}{12n}} = 45.5 - 2.34 * \sqrt{\frac{(90-30)(90+1)}{12*30}} = 36.45$$

$$CL = \bar{R}_{..} = 45.5$$

$$UDL_{asymptotic} = \bar{R}_{..} + \omega(\propto; J)\sqrt{\frac{(N-n)(N+1)}{12n}} = 45.5 + 2.34 * \sqrt{\frac{(90-30)(90+1)}{12*30}} = 54.65\ .$$

Step 3. Draw the ANOMR chart: Draw three horizontal lines for $LDL_{asymptotic} = 36.45$, $CL = 45.5$ and $UDL_{asymptotic} = 54.65$ on a graph paper. Then mark the rank means $\bar{R}_{.1} = 37.55, \bar{R}_{.2} = 42.4, \bar{R}_{.3} = 56.55$ on the same graph. Of course we can use a computer package, such as Excel, to produce a nice and accurate chart. Fig. 4 displays the resulting chart for the modified ANOMR.

Table 6. Ranks of the Three Contractors in the Combined Sample

	Contractor A	Contractor B	Contractor C
	31.5	5	69.5
	42.5	42.5	75.5
	17.5	13.5	17.5
	37	37	67.5
	37	37	74
	47	37	47
	22	3	69.5
	17.5	8	72
	31.5	86	31.5
	51	85	79.5
	60	67.5	1
	57	57	57
	17.5	87	82.5
	53.5	11.5	53.5
	63.5	63.5	63.5
	37	13.5	79.5
	25.5	90	25.5
	25.5	25.5	6.5
	22	22	63.5
	51	2	51
	47	6.5	79.5
	42.5	10	72
	31.5	89	75.5
	28.5	4	17.5
	11.5	88	17.5
	28.5	9	77
	47	72	47
	37	63.5	82.5
	42.5	79.5	84
	63.5	57	57
Rank Total $R_{.j}$	1126.5	1272.00	1696.50
Rank Mean $\bar{R}_{.j}$	37.55	42.40	56.55
Grand Rank Mean $\bar{R}_{..}$	45.50	45.50	45.50

Step 4. Interpret the chart and make conclusion: The ANOMR chart in Fig. 4 shows that the rank mean 56.55 of contractor C falls above the asymptotic UDL (=54.65) while contractors A and B rank means fall within the decision lines. Therefore, ANOMR concludes that Contractor C is significantly different from the other two contractors. This conclusion contradicts the parametric ANOVA and ANOM conclusions, but it is consistent with the descriptive analysis findings of Section 2 that the contractors differ in more than one aspect. The fact that ANOMR does not require the data to be normally distributed, makes its conclusion more trustworthy than the conclusions of ANOVA and ANOM.

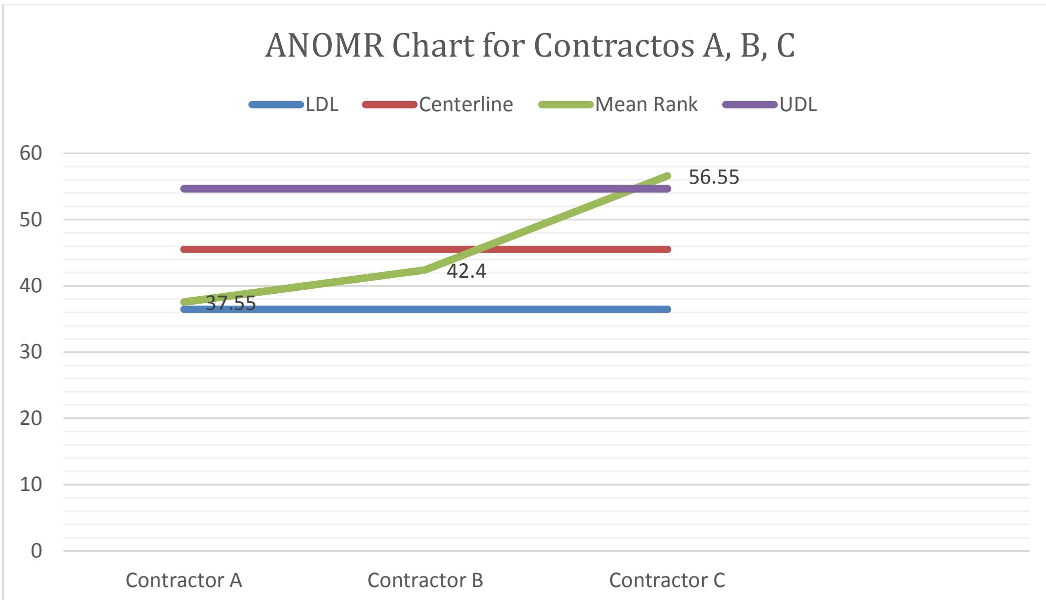


Fig. 4. ANOMR chart for the rank means of contractors A, B, and C.

6 Kruskal-Wallis (K-W) Test

The K-W test procedure is a nonparametric procedure for testing the equality of several population centers (the hypotheses in Formula 4) in a one-way lay out model; it assumes that data represent independent random samples drawn from continuous (not necessarily normal) populations having the same variance. K-W test has useful applications in management and marketing as in Akdeniz et al. [14].

To test for a significant difference among several population centers, the K-W test replaces each data value y_{ij} by its rank R_{ij} (Formula (9)), in the combined sample. Referring to Hollander and Wolf [15], the K-W test statistic is denoted by H and has the formula:

$$H = \left(\frac{12}{N(N+1)} \sum_{j=1}^{J} \frac{R_{.j}^2}{n_j}\right) - 3(N+1)\,, \quad (22)$$

where $R_{.j} = \sum_{i=1}^{n_j} R_{ij}$ is the rank total of the jth sample.

Using the rank totals in Table 6, we calculate the K-W test statistic H:

$$H = \left[\frac{12}{90(90+1)}\left(\frac{1126.5^2}{30} + \frac{1272^2}{30} + \frac{1696.5^2}{30}\right)\right] - 3(90+1) = 8.5677.$$

The test statistic $H = 8.5677$ with a *P-value* of 0.0138 (as delivered by SPSS) leads to concluding that the three contractors are significantly different. This conclusion is consistent with the ANOMR conclusion in Section 5 and with the descriptive statistics findings in Section 2; however, it contradicts the parametric ANOVA and the ANOM conclusions.

7 Summary and Conclusions

This paper modifies Analysis of Means by Ranks (ANOMR) and applies it to a case study in contract acquisition analysis.

The modified ANOMR is applied to data that involves the analysis of contract proposals. For comparison purposes, the same data are analyzed by some well-known parametric and nonparametric tests. The parametric tests are ANOVA and ANOM; the nonparametric tests are ANOMR and the K-W. The data used appears in the Defense Acquisition University (DAU) class CON 270 case study that involves comparing three contract proposals.

Results in this paper, show that the parametric test procedures (ANOVA and ANOM) do not detect a significant difference among the three contract proposals, while the nonparametric procedures do. The parametric conclusions are doubtful because a descriptive statistics analysis (skewness, kurtosis, histograms) indicates that the required parametric assumption of normality is not satisfied. The nonparametric procedures (ANOMR and K-W) do detect a significant difference among the contractors; this conclusion is more trustful because the assumption of normality is not required. Additionally, ANOMR has the advantage of producing charts with decision lines that can pinpoint which population is significantly different from the rest.

Granted that a savvy researcher may advocate the use of a normality test (e.g., Anderson-Darling test) and then use an appropriate transformation to bring the data closer to normality. However, data transformation is usually cumbersome because one has to try several types of transformations (square root, logarithmic, etc.) before being successful. Even with a successful transformation, it is usually difficult or confusing to interpret the conclusions in term of the original population parameters. Actually, colleges (even statistics departments) rarely expose their students to tests for normality or data transformation topics.

Using parametric (ANOVA and ANOM) test procedures without a preliminary examination of the probability distributions of the data could lead to misleading conclusions. Nonparametric (ANOMR and K-W) test procedures are robust to the shape of the distribution and generally produce more reliable conclusions. Therefore, we advocate that it is easier and more reliable to use nonparametric tests routinely.

Competing Interests

Author has declared that no competing interests exist.

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